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OPTIMUM SIGNAL RECEPTION AND THE POTENTIAL ACCURACY OF SPACE MEASURING COMPLEXES

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PREFACE

On October 4, 1957, the Soviet Union launched the first artificial earth satellite in the world, laying the developmental groundwork for aerospace technology. This has already given mankind many important discoveries with many important practical applications.

As we know, the enormous diversity of tasks in space has already required the launching of hundreds of artificial Earth satellites and dozens of space vehicles to the Moon and other planets in the Solar System. In all cases, we must be able to solve radar location problems for the space vehicle, or measure its parameters of motion (its orbital parameters) and to determine the coordinates or parameters of motion of other objects.

Determination of parameters of motion is ordinarily carried out in two stages, tantamount to "radiotechnical" and "ballistic" parts of the problem. The so-called first processing stage is where signals received by the radar equipment undergo optimum processing in terms of space vehicle [SV] coordinate determination and their derivatives with respect to the radar device. The second stage is where the SV parameters of motion (orbital parameters) are determined and given prognosis in terms of these data using a computerized stellar mechanics device.

P. Olyanyuk's book is interesting in that the direct relationships of unknown parameters of SV motion are formulated as a function of received signal structure (these signals being signals with regularly varying parameters) and the potential accuracy of radio-technical measuring units is defined. The author devotes particular attention to the relatively great determinacy of motion of many SV and thus the comparatively great correlation time of motion parameter fluctuations. This all allows a prolonged accumulation of signal and consequently, one may increase accuracy of measurement for parameters of motion. The generalized autocorrelation functions which the author logically introduces for signals with regularly varying parameters let us directly evaluate the ponderability of apriori data and accuracy of the measuring means.

A similar approach, though remaining within the framework of the accepted theory of statistical solutions, may also be of interest from the methodologic point of view, in that it lets us clearly evaluate the accuracy of various radiotechnical units and synthesize their optimum design. The cited material is sufficient to aid the reader in finding concrete applications for the suggested methodology.

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LIST OF SYMBOLS AND NOTATIONS

A	amplitude of received signal
A_r	amplitude of reference signal, shaped at receiver in terms of apriori data
A_m	amplitude of modulated carrier
a	semimajor axis of Keplerian ellipse
a_E	equatorial radius of adopted reference ellipsoid of the Earth
B, B_a	correlation matrices of measurement error in apriori data
B_g	geodetic latitude and its rate of change
D	length of side of square antenna or diameter of round antenna
E	eccentric anomaly
e	eccentricity of Keplerian ellipse
e_E	eccentricity of reference ellipsoid of Earth
F	Dopplerian frequency drift
f	frequency of carrier oscillation
g	vector of parameters of motion of space vehicle in geocentric rectangular system of coordinates
g_1, g_2	its coordinates and velocity components
H	altitude with respect to surface of adopted reference ellipsoid
I_ξ	informativeness of trajectory with respect to some quantity ξ
$I_0(x)$	zero-order Bessel function of imaginary reasoning
i	angle of orbital inclination
J	transform matrix
J_r, J_c, J_s, Jg	transform matrix of differentials in transition from rectangular, cylindrical, spherical, and geodetic system to initial rectangular system
J_{ξ_q}	Jacobi transition matrix from coordinate ξ to coordinate g

k — wave number

L — geodetic longitude

M_0 — mean anomaly at time t_0

m — height of amplitude modulation; number of defined parameters of motion

N — radius of curvature of first vertical circle on surface of reference ellipsoid at point of observation

N_0 — spectral density of interference

$N(t,r)$ — complex amplitude of interference

$n(t,r)$ — instantaneous value of interference

P, P_k, P_j — transition matrix from differentials of Keplerian parameters to differentials of initial conditions of motion in an inertial rectangular system of coordinates, its k^{th} row and j^{th} column (Chapter VI); radiated power of on-board transmitter (Chapter V).

p — parameter of Keplerian ellipse

Q_c, Q_s, Q_g — vectors of linear components of coordinates of cylindrical, spherical, and geodetic systems of reference

q, q_a — vectors of real and apriori values of parameters of motion, vector of Keplerian parameters

Δq — vector of difference between actual and apriori values of parameters of motion

q_p — vector of undefined parameters of motion

q_c, q_s, q_g — vectors of parameters of motion in cylindrical, spherical, and geodetic coordinate systems

R_c, R_s, R_g — matrix of revolution in transition from differentials of cylindrical, spherical, and geodetic systems of reference to differentials of rectangular coordinates

$R_z(\lambda)$ — matrix of revolution, describing rotation of rectangular system of coodrinates around axis z at angle λ

$r(t)$, $r_a(t)$ instantaneous distance between observer and space vehicle and its apriori value

r_E, r_{Za} geocentric radius-vector of space vehicle and its apriori value

r_K, r_{Ka} geocentric radius-vector of space vehicle and its apriori value

r_C linear coordinate of a spherical system

r_{Ec}, r_{Eca} geocentric radius-vector of center of antenna and its apriori value

r_A radius-vector of instantaneous point of antenna

S active surface of receiving antenna

s(t,r) instantaneous value of signal

T time of observation of duration of measurement

t, t(0) instantaneous time and some defined value of it

U_c, U_s, U_g matrix of covariance of differentials of velocity components of rectangular system of reference with differentials of components of cylindrical, spherical, and geodetic systems

u angle of latitude

V volume of space occupied by elements of receiving antenna

V_c, V_s, V_g matrix of covariance of differentials of velocity components of cylindrical, spherical, and geodetic systems of reference with differentials of components of a rectangular system

v velocity of space vehicle

υ_{gr}, υ_{ph} group and phase rates of propagation of radiowaves

W_c, W_s, W_g matrices of direct transformation of differentials of coordinate components in transition from cylindrical, spherical, and geodetic systems of reference to rectangular

W_K matrix of direct transformation of differentials in transition from Keplerian elements to initial conditions of motion in a geocentric inertial rectangular system of reference

W_i matrix-squares forming the matrix W_k, $i = 1,2,3,4$

$w(x)$ probability density of a random quantity

x, y, z rectangular coordinates

$x = ||x_1 x_2 x_3||^t$ topocentric radius-vector of space vehicle

$x = \upsilon T/\rho$ generalized coordinate (Chapter V)

$Y(t,r)$ complex amplitude of a constructive mixture of signal and noise

$y(t,r)$ instantaneous value of signal-and-noise mixture

Z space-time autocorrelation function (ACF) of signal field

Z_T autocorrelation function of fluctuations in the period of revolution of an artificial earth satellite

$Z_{T'}$ autocorrelation function of fluctuations in the rate of change of periods of revolution of an artificial earth satellite

z z-coordinate of cylindrical system of reference

E signal energy

E_k signal energy expended during k^{th} interval of correlation of fluctuations of initial phase

α vector of signal parameters

β, β vector of random signal parameters, initial phase

γ, γ_a, $\Delta\gamma$ angular topocentric coordinate of space vehicle, its apriori value and the difference between them

θ actual anomaly

$\kappa = k^2 PS/8\pi$ generalized parameter

λ wave length of oscillation carrier

λ_c, λ_s longitude in cylindrical and spherical system of reference

μ gravitational constant of Earth

ξ, η, ζ cartesian geocentric or topocentric coordinates of space vehicle at some point in time (initial conditions of motion)

$\xi_1 = \xi,\ \xi_2 = \eta,\ \xi_3 = \zeta,\ \xi_4 = \xi,\ \xi_5 = \eta,\ \xi_6 = \zeta$

ρ	traverse distance (Chapter V), second linear coordinate of cylindrical system of reference (Chapter VI)
τ	time lag (Chapter II), moment of transit of perigee (Chapter VI)
ϕ	law of signal phase modulation
ϕ_i	phase of interference
Ω	frequency of modulation, longitude of ascending node
ω	signal frequency; angular distance of perigee
$\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3$	Jacobi elements
H, g, h, L, G, l	canonical elements
$\rho_2\ \omega_1\ \omega_2\ \ L, \rho_1, \lambda$	first system of Poincare elements
$\xi_2\ \eta_1\ \xi_1\ \ L, \eta_2, \lambda$	second system of Poincare elements
$g_1,\ h_1,\ M_2,\ P_1,\ K_1$	orbital elements similar to Keplerian
$a,\ \Omega,\ i,\ K,\ M_1,\ h$	
$a,\ \Omega,\ \cos i,\ K,\ M_1,\ h$	

INTRODUCTION

This book treats questions on the theory of signal processing in aerospace measuring units, which include orbital measuring units, terrestrial and orbital navigation by satellite, and space geodetic units. Aerospace measuring units have a number of specific features, such as the following:

1. They are designed to determine the parameters of motion of objects whose trajectory has significant determinacy, resulting from the relatively small number of random disturbances affecting them. Research has shown that the duration of the parameter fluctuation interval of orbits induced by variations in atmospheric density is on the order of a day [30]. Space measuring units greatly differ from radar units in this respect. Radar is designed to determine the parameters of motion of objects travelling in the atmosphere. The correlation interval of a random velocity component, in the latter case, is on the order of seconds or minutes.

2. The great quantity of the measurement process lifetime is the result of space vehicle trajectory determinacy. We know that the overall duration of this process may be several hours: measurements may be made during the indicated time or in short spans of time which are not contiguous but fall within the limits of the orbital parameter fluctuation correlation interval.

3. A particular feature of aerospace radiotechnical units is the great dispersion of measuring means in space. In spite of this, with accurate synchronization of the work of individual telemetry units, the aerospace unit as a measuring system is a unified entity.

4. To measure the parameters of motion of space vehicles, we may use both short pulsed signals, whose phase fluctuation correlation interval is small, and long continuous signals whose phase fluctuation correlation interval may be extremely great. Continuous emission is inherent in phase and Doppler systems of measurement and permits us to produce signals of high energy with comparatively low radiation power. The development of continuous emission systems was promoted by successes in the field of signal generation (high frequency stability) and achievements in some other fields of modern radioelectronics. In using a continuous emission system we must deal with the fact that signal parameters carrying useful information fluctuate within wide limits during the period of measurement.

5. Aerospace measuring units may be used for direct measurement of instantaneous distances, angles and their derivatives. The final goal of measurement, however, is to determine parameters of motion. These may be initial values of coordinates and velo-

city, Keplerian elements of motion, navigation, geodetic and geophysical parameters. A typical feature of parameters of motion is that their constancy interval significantly exceeds the constancy interval of topocentric coordinates and velocity.

6. Parameters of motion are determined under conditions of interference, which are theoretically nonremovable inteferences of the fluctuation type.

All these features are intrinsic to orbital measurement units and satellite navigation and geodesy systems as well.

The theory of aerospace measuring units, which units the theory of radiotechnical measurement methods for parameters of motion and the theory of determining orbits, has now been rather thoroughly developed.

The primary content of the theory of radiotechnical methods of parameter of motion measurement constitutes problems of isolating signals under the influence of random disturbances. The modern state of the theory of methods of signal isolation on the background of fluctuation interference may be described in the following manner. Methods have been developed to isolate signals which are purely random processes and have instantaneous values which are of random magnitude, characterized by certain laws of distribution. This type of signal is encountered in automatic control systems, in control systems, in data transmission analog systems, and so forth. Therein, they represent useful information and yield to the most possibly accurate reproduction. We are indebted to N. Viner and his followers for the establishment of this. The mathematical fonndations of the theory were lain by the fundamental work of A. N. Kolmogorov, N. Viner, R. Ye. Kalman and others [12, 29].

On the other hand, another branch of the theory of signal filtration has developed in response to the needs of radar and radio-navigation. The basic content of this theory is the isolation of regular signals having random parameters. As we know, radar and digital systems of communication use modulated and unmodulated signals of the harmonic type, whose individual parameters (amplitude, frequency, phase, location in time) are used to represent useful information and are by nature random.

The most characteristic task in this case is the isolation of signals having random parameters, whose magnitude is kept constant during the measurement process. Since we know the nature of the signal being received, the filtration process consists only in determining the informative parameters of the signal, and not in reproducing the shape of the signal which, in the receiving process, may become extremely distorted.

The mathematical foundation of the theory of isolation of regular signals having constant random parameters is composed of the theory of evaluating distributive parameters of random quantities (or processes), which represents an important division of modern mathematical statistics. The mathematical apparatus of the theory begins with Gauss, but it received its further development in the last few decades in the works of R. Fischer, G. Kramer, Yu. Linnik and other mathematicians, as well as in the works of V. Kotel'nikov [10], F. Woodword [5], V. Siforov, Ya. Shirman and other radio specialists.

The development of the theory of signal isolation is now undergoing further development.

From the evaluation of one or two informative signal parameters (the most practical interest is offered by such parameters as signal lag time, which carries information about range, and frequency, which describes the velocity of the object and represents a linear term of phase lag in a Taylor expansion)we have come to the task of evaluating a larger number of parameters. The number of additionally defined useful signal parameters included, specifically, the second and higher derivatives of range. These questions were developed in the studies of Ye. Kelly, R. Vishner, S. I. Krasnogorov and others. The evaluation of the magnitude of derivatives of high orders makes possible a more thorough description of motion and a more accurate reproduction of the law of motion of an aircraft.

Another trend in the development of a theory of signal isolation is associated with the consideration not only of time, but also space properties of signals. If the signal is initially considered only as a process developing in time, and filtration has been reduced to consideration of just time or spectral distinctions of the signal and interference, then we would now have in mind both time and space properties of the useful and interfering electromagnetic fields in a defined area of space. This approach is not only associated with the fuller utilization of information, but also with a different solution of the filtration problem which allows the simultaneous determination of range and velocity, and the angular coordinates of objects and their derivatives as well. In terms of the results of repeated measurement, the space-time filter makes it optimally possible to determine the position of an object in space and its rate of travel. Various aspects of the theory of space-time filtration of signals were developed in the work of R. Bracewell, G. Urkowitz, S. Fal'kovich [23] and others.

The operations performed on signals in the process of determining the topocentric coordinates and velocity are sometimes called primary processing.

The second constituent part of the theory of measurement of parameters of space vehicle motion embraces questions of secondary processing of orbital information, which comprises the basic content of the theory of determination of orbits.

In its most general aspects, the task of determining orbit includes the determination of some set of parameters which unambiguously describe the motion of a space object, in terms of data obtained in measuring geometric and kinematic quantities connected with the parameters to be determined. These parameters are determined by functional relationships. In processing measurement data, we achieve a union of measurement data and an attenuation of random error effects. The mathematical foundation of the theory of orbital determination is composed of a stellar mechanics device and the statistical theory of evaluation of distributive parameters, which was already mentioned. In this regard, the most practical application was produced by the method of least squares developed by K. Gauss. A rather complete presentation of the possibilities of the theory of orbital determination as it applies to problems of trajectory determination of space vehicles is given in the famous studies of P. El'yasberg, V. Yastrebov, E. Akim, T. Eneyev, and others.

Among those methods of data processing which have been developed in recent years, we should note the dynamic filtration method developed by R. Bettin, which encompasses the conditions of data processing realized in proportion to the access of measurement data and proposing the use, in each subsequent stage of analysis, of the results of trajectory determination from the preceeding stages. This method enables us to increase the operational character of data output on the orbit.

The existing theory of space measurement units ensures the solution of the basic problems facing them, and makes it possible to plan and make use of high-accuracy units of various design. Several limitations are intrinsic to it, however, and therefore it does not fully and in all cases permit us to study the laws of operation of units which, as we know, are of great complexity.

One of the main shortcomings of the existing theory is the absence of an organic unity of its constituent parts, which leaves us with an impression of its imperfection. The division of the theory of units into constituent parts was put together historically and reflects most organizationally and technically expedient division of the process of determining parameters of motion into processes of measuring topocentric coordinates and processing the data of these measurements. Of course, in the analysis of unit operation and a mathematical description of the processes, the examination of questions of topocentric coordinates and measurement analysis was done in separate parts which, in most cases, is fully justifiable.

This sort of division may sometimes become a restricting factor. In a framework of an individual examination of "radio" and "ballistic" problems, it becomes impossible to obtain clear and sufficiently complete answers to a number of questions which arise in the selection of efficient specifications for the units. There are, as we know, interrelated properties of the units which are of diverse nature, particularly the characteristics of signals (their power, duration and spectral makeup), structural characteristics (composition of the unit, quantity and type of directly measured parameters), characteristics of a geometric nature (juxtaposition of systems on the Earth's surface). In the traditional approach, it is difficult to evaluate the potential accuracy of space measuring devices. The comparison of the potential resources of the devices which differ in parameters to be measured and the study of several other questions is difficult as well.

The selection of the basic characteristics of complexes is extremely complex and requires the consideration of numerous factors of various type. The mere listing of those quantities on which an orbital measurement complex depends is enough to convince us of this.

The accuracy of a single measurement of range and angular coordinates is defined by the energy, frequency or band width of the signal received. In turn, the energy of the signal at the point of reception depends on the distance between the observer and the space vehicle and thus, on orbital parameters, coordinates of the receiving point, and the time of measurement.

Orbital determination is equivalent to determination of the spatial coordinates and velocity vector of the SV. In view of the fact that the accuracy of angular determination is not always sufficient, range and Doppler methods of measurement have received the widest use, where we have recourse to repeated simultaneous or nonsimultaneous observations from several points on the Earth's surface. In this connection, orbital determination accuracy is a function not only of the accuracy of individual measurements, but also of the juxtaposition of ground stations, orbital parameters, the choice of trajectory segments, and so forth.

Therefore, in selecting efficient characteristics for the entire measuring complex and its individual components, we must also take account of signal properties and purely geometric factors. The manner in which these laws develop their mechanisms of effect requires that the complex be considered a unified space-time measurement system.

The problem of evaluating potential accuracy is closely linked with the problem of selecting efficient characteristics for space measurement complexes.

Potential accuracy is the customary designation for the greatest accuracy which may be attained in measurements using a measuring system which is free of inherent error; the system uses a certain signal which provides metric information under conditions of fully defined disturbances. In other words, potential accuracy is the accuracy attained without instrumental errors with the optimum reception of a useful signal against a background of interference.

The information and reference electromagnetic fields in a given area of space at a given interval of time serve as the signals in space measuring complexes. Interference is the noise electromagnetic field which often may be a random field of the fluctuation type. As the potential accuracy of the space measuring complex we will understand the limiting accuracy of determination of parameters of motion (orbital parameters in particular) which is achieved in the most advantageous utilization of the fields indicated. We can see from this definition that since quantities determined with the aid of space measuring complexes are parameters of motion (especially in orbital parameter trajectory measurement complexes) and an electromagnetic field is the signal, the evaluation of potential accuracy should also be made with a provision for the radiotechnical and ballistic aspects of the problem.

On the other hand, we know that to determine some of the same parameters of motion, we may use the electromagnetic field in a different manner. Useful information on motion may generally be included in several different field parameters (e.g., in fluctuations of amplitude and frequency of received signal). There is, of course, no reason to anticipate an identity of measurement results when different metric data sources are used. For that reason, a comparative evaluation of potential feasibility of radiotechnical measurement methods differing in the type of information parameter being used is of interest to us. In other words, along with the problem of evaluating the potential accuracy of measurements which describe the potential resources of the field as a whole, we must evaluate the potential accuracy of measurements with the addition of various field parameters, i.e., the task of evaluating potential feasibilities of diverse methods of measurement.

It is worthwhile to compare such space measurement methods as the telemetry and Doppler methods. We should mention that a comparative evaluation of the potential accuracy of the Doppler and other known methods of measuring *the rate* of motion does not encounter any problems. But a strictly comparative analysis of the

Doppler and, let us say, the phase telemetry methods of *coordinate* determination can not be done in the ordinary approach: the comparison of these methods is impossible in terms of single measurements of frequency and phase. A comparative evaluation of these methods in the stage of data processing, without allowance for energy and properties of the signal is not correct enough. The sole approach to the measurement problem, which was discussed above, also happens to be rather attractive in this instance. The potential accuracy, thus, is an important technical characteristic of the measuring complex, reflecting the ultimate resources of the complex as a unified measurement system.

In arranging the methods for evaluating potential accuracy, it is possible to evaluate the degree of perfection of various types of measuring devices, to make note of ways of improving them, to define the degree of perfection of methods and means used to isolate signals from interference, and to state recommendations for means of improving the methods.

We know that the current theory of optimum signal filtration and evaluation of potential accuracy of radio-navigation and radar systems has been extremely fruitful, and has had not only purely theoretical but also a rather great applied value.

This study will attempt to state the basic questions of the theory of space measurement complexes which is founded on the consistent application of methods of the theory of statistical solutions and consideration of trajectory determinacy as constituting one of the basic features of the measurement process. The process of measuring parameters of motion, which includes both primary and secondary signal processing, can be seen as a unified process whose goal it the determination of orbital parameters, navigational or geodetic quantities, i.e., "secondary" parameters of mtoion.

In turn, a measurement complex which is made up of a large number of measurement means, concentrated in space and functioning in coinciding and non-coinciding segments of time of different durations, can be represented as a unified system, implementing space-time filtration of signals for direct determination of the previously noted "secondary" parameters of motion.

It may generally be stated that this theory is some generalization of the current theory of space measurement complexes for signals whose total duration equals that of the fluctuation correlation interval of the parameters of motion. Within the framework of the given generalization, we must view the processes of primary and secondary signal processing from unified positions of the theory of statistical solutions and development of methods for evaluating potential accuracy of space measurement complexes of various types, including complexes which contain Doppler systems of space trajectory determination or navigation and geodetic parameters.

The book contains an introduction and six chapters.

The first chapter describes the basic working model of the signal -- a signal with regularly varying parameters -- and reveals the basic properties of signal and interference fields affecting space radiotechnical complexes.

The second chapter is devoted to methods of direct evaluation of the parameters of motion in terms of the signal acting in a gien field of space. Therein are cited algorithms for optimum filtration of signals with regularly varying parameters acting against a background of an additive random interference field; and the properties of the autocorrelation function of the auxiliary signal field are studied.

The third chapter contains an analysis of evaluative accuracy for parameters of motion of SV with optimum signal processing; analytical expressions are given for the maximum value of the secondary derivatives of autocorrelation functions of the signal field, characterizing the potential accuracy of measurements with complete utilization of data resources of the signal's electromagnetic field.

The fourth chapter is devoted to an analysis of the potential accuracy of individual methods of measurement -- phase and pulse telemetry, Doppler, and goniometric. Therein are cited examples of a phase telemetry system, whose principle of action is close to the optimum of the planetary radar system of the Academy of Sciences of the USSR.

The fifth chapter contains a discussion of potential accuracy of determination of various systems of parameters of motion. An attempt is made therein to divide the evaluative process into two independent parts: the measurement process and the coordinate transformation process. The chapter examines several properties of coordinate transformations and an example is given of evaluation of potential accuracy of the telemetry and Doppler methods of measuring SV parameters of motion in one pass over the field of vision. Data are then given on the informativeness of various segments of the measured trajectory obtained through the use of the research method presented in this study.

The sixth chapter examines the properties of matrices of the basic coordinate transformations used in determining orbital parameters. Methods are given here for computing the transition matric and formulary relationships for the most common coordinate transforms. Chapter Six was written by V. I. Mikhaylik.

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OPTIMUM SIGNAL RECEPTION AND THE POTENTIAL ACCURACY OF SPACE MEASURING COMPLEXES

P. Olyanyuk

Chapter 1 /19*

BASIC FEATURES OF SIGNALS AND INTERFERENCE IN SPACE MEASURING COMPLEXES

1. Parameters of Motion

The parameters of motion are constant quantities which unambiguously describe the law of motion of an object in a given interval of time. The number and physical meaning of the parameters of motion are defined in terms of the mangitude of forces which give rise to motion, their nature, and the inertial properties of the object.

The flight of an airplane in the atmosphere, the motion of a ship on the seas, the movement of an automobile on the ground -- these all result from forces which, in addition to regular constituents, have constituents which are random in nature and are very great and rapidly fluctuating in magnitude and direction. The fluctuation rate of random quantities, as we know, may be ascertained in terms of the magnitude of the correlation interval. The duration of the correlation interval of fluctuation rates of ground and surface objects is not great, usually not in excess of several or tens of seconds.

Which parameters should be used to describe the motion of such objects? Toward that end, it would seem we must use the values of coordinates and velocity which are related to points in time which are separated from one another by a quantity on the order of the correlation interval duration. Higher derivatives of the coordinates in time may be used for a more precise description of motion.

The law of motion of terrestrial objects may be defined using radar-equipment methods by the simultaneous measurement of distances from several ground points or from one range and its angles. We may also measure the time derivatives of these quantities, usually being limited to a definition of the first derivatives.

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* Numbers in the margin indicate pagination in the foreign text.

The duration of measurements is small: it should not exceed the duration of the correlation interval of parameters of motion.

Due to the variability of the magnitude and direction of velocity, the prediction of motion is accompanied by great errors. For that reason, apriori data on motion is of low accuracy and is not, as a rule, utilized in the measurement process. /20

The flight of a space vehicle (SV) is chiefly affected by the gravitational field and the forces generated by its engines. These forces are regular in nature. In addition to effects of a regular nature, the motion of objects in space is also affected by random disturbances which are comparatively small. Therefore, the duration of the correlation interval of velocity fluctuation of the SV exceeds a similar quantitiy for earth-bound objects by several orders and may go as high as many hours or days. The motion of a SV, thus, is almost completely determinant in nature.

The flight of a space vehicle is also defined by coordinates and velocity which are related to some specified point in time within the correlation interval. But since the duration of the correlation interval of SV velocity fluctuation may be measured in hours and days, as was mentioned, the motion of a space vehicle is usually described by only six parameters for the entire interval of operation of the object or for a significant part of this interval. As was stated, in similar stages of operation of terrestrial objects, in terms of duration, we must include a large number of similar six-element parameter groups.

SV coordinates and velocity at a specific moment in time are often called the initial conditions of motion, because they are constant quantities defined in the process of integrating equations of SV motion. In SV motion in a central gravitational field, the role of parameters of motion may be filled by Keplerian orbital parameters or some other sets of geometric and kinematic quantities. The motion of objects is a gravitational field of a more complex structure is described by osculating orbital elements. SV parameters of motion are also called orbital parameters. The duration of the process of determining SV parameters of motion may be many hours.

The direct determination of orbital parameters is, as a rule, impossible. Measurements are made of the range from earth-bound points, angular topocentric coordinates of the space vehicle, radial and angular velocities, and the orbital parameters are defined by processing measurement data in the computer. /21

Under determinant motion conditions, instead of a simultaneous measurement of ranges and radial velocity constituents with respect to *three* spaced points, we may restrict ourselves to the measurement of ranges and the corresponding velocity constituents from *one* ground point. In this connection, measurements from one point are only required to be made in specified time intervals. In order to reduce random error, the number of measurements is usually large to reduce their influence on the results of the measurements.

A distinctive feature of the parameters of motion, i.e., those quantities which are the final goal of the measurement process, is their constancy over the entire interval of measurement. Directly measured quantities within this interval typically fluctuate quite rapidly and within wide ranges. In the rapid fluctuation of measured quantities, the measurement process appears to become more complicated. The quality of quantities measured under similar conditions, as a rule, is lower than the quality of measured quantities which fluctuate slowly. We may theoretically free ourselves from the rapid fluctuation of measured topocentric coordinates if we measure the slowly fluctuating deviations of defined coordinates from their predicted values instead of measuring rapidly fluctuating instantaneous topocentric coordinates.

In orbital measurement systems, the implementation of such a procedure may be done in practice, because the flight of objects in outer space has a higher degree of determinacy than the motion of terrestrial objects. By using deviation measurements in place of the theoretical values, random errors may be leveled out.

Therefore, in determining SV parameters of motion, we may adduce apriori data on the parameters of motion. These data are, of course, not accurate enough. For that reason, apriori values of the parameters of motion are usually represented as random quantities, described by fixed laws of distribution. Nonetheless, the use of these data greatly simplifies the solution of the task of determining the parameters of motion which, under these conditions, would involve the problem of adding accuracy to the apriori data.

In examining the overall picture of procedures for determining /22
SV parameters of motion, a description of the entire mechanism of determination of these constants must be used which would not require the use of rapidly fluctuating topocentric coordinates; and in which the entire procedure of determining the orbital parameters would constitute the one and only measuring procedure. In undertaking to solve such a problem, of course, we must realize that there is no basis for considering a similar unified approach as a practically expedient procedure for processing signals to replace those procedures currently in use. Moreover, the division of the orbital determination procedure into technically uncorrelated operations is simply obligatory in the majortiy of applied cases. In theoretical analysis, however, the "unified" approach

may prove useful, since it possesses a minimum amount of initial restrictions and is free of any provisional agreements on the measurement methods. These features of the given research method enable us to rather strictly examine the question of the potential resources of space measuring devices and a number of allied problems.

1.2. Some Data on the Fluctuation of the Parameters of Motion of Space Vehicles

The preceeding section stated that SV parameters of motion were considered as random quantities, invariable during the period of observation, defined by set laws of distribution.

In reality, however, the parameters of SV motion are variable, and in the course of time we observe secular, periodic and random fluctuations in these quantities. Secular and periodic fluctuations are induced by disturbances of a regular nature. Random fluctuations of the parameters occur under the influence of forces which fluctuate in accordance with a random law, first among which is the force of aerodynamic resistance.

Secular and periodic fluctuations of the parameters of motion may serves as a source of data on the structure of the gravitational field and are taken into account in processing measurement results. Random fluctuations are a source of error in determining orbital parameters and impose certain limitations on the method of measurement.

Therefore, in a more precise examination , SV parameters of /23
motion are random processes, whose mathematical expectation has a secular and periodic variation. In order to properly interpret measurement results, we must understand the basic statistical characteristics of these processes and, in particular, a quantity such as the correlation interval duration. The significance of this parameter of the random process is due to the fact that it defines the permissible duration of the measurement process.

Some data have recently appeared in the literature [8, 9, 30] which describe the duration of the correlation interval of SV orbital parameters which are subject to the retarding action of the terrestrial atmosphere on their motion. Unfortunately, the amount of experimental data accumulated by research is still small and they are only related to a limited range of conditions. These data, nonetheless, let us form some idea on the order of magnitude of the correlation interval. In particular, in study [30], the results are given for orbital acceleration of the artificial Earth satellites "Explorer I" and "Explorer IX". Orbital acceleration implies the rate of fluctuation in the period of revolution of the satellite, which describes the action of aerodynamic resistance.

Table 1.1 gives orbital parameters for the Explorer I and Explorer IX satellites and some other data.

TABLE 1.1. DATA ON EXPLORER I AND EXPLORER IX

Parameters	Explorer I	Explorer IX
orbital inclination, degrees	33°,2	38°,86
height of perigee, km	357	634
height of apogee, km	2,562	2,583
eccentricity	0.141	0.121
initial period of revolution, min.	114.8	118.28
area of cross section, m^2	0.26	10.8
weight, kg	14	6.63
Launch data	2/1/58	2/16/61

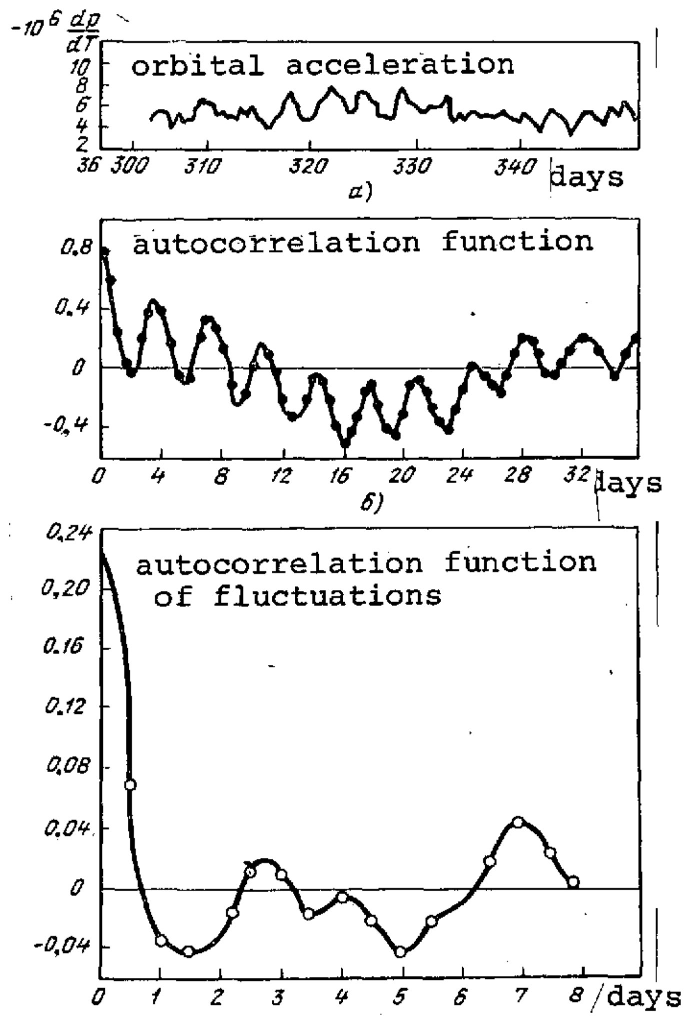

Fig. 1.1. Orbital acceleration (a),its autocorrelation function (b), and autocorrelation function of fluctuations (c) of Explorer I, in elliptical orbit, perigee of 350 km.

Figures 1.1 and 1.2 give the results of orbital acceleration and autocorrelation functions of orbital acceleration for these satellites.

Figure 1.1a illustrates /24
the relationship of Explorer I's orbital acceleration as a function of time; Fig. 1.1B illustrates the autocorrelation function of this process. From Fig. 1.1C we can see that the autocorrelation function is a superposition of three functions. The first two functions are periodic in nat- /25
ure; the third is aperiodic. The twenty-seven day period of fluctuation of the first constituent coincides with the period of the Sun's rotation on its axis and describes the effect of solar activity on processes occurring in the Earth's atmosphere. The four day fluctuation period of the second constituent is defined by the precession of the vector of the satellite's angular

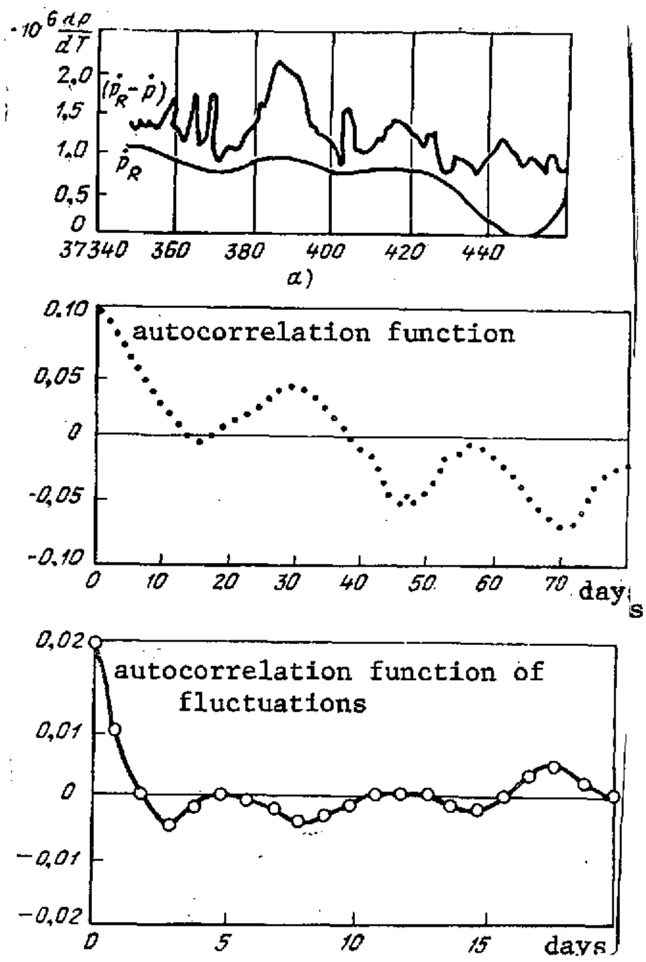

Fig. 1.2. Relationship of atmospheric resistance $(\dot{P}_R-\dot{P})$ and light pressure $(\dot{P}_R)$ as functions of time (a); autocorrelation functions of orbital acceleration (b) and its fluctuations (c) for Explorer IX, elliptical orbit, height of perigee 630 km.

momentum of revolution.

The third, aperiodic constituent, obtained after subtraction from the autocorrelation function shown in Fig. 1.1B, of periodic constituents, is shown in Fig. 1.1C. It describes the fluctuation of orbital parameters. It follows from Fig. 1.1C that the fluctuation correlation interval comprises a quantity somewhat less than several days. /26

Cited in Fig. 1.2 are similar data obtained in processing results of observation of Explorer IX. This satellite was a gas-filled balloon and thus its rotation about its axis was not accompanied by fluctuation in the force of resistance. However, due to the large area of cross section and the low mass of the satellite, its motion was noticeably affected by the force of light pressure, whose fluctuation rate is shown in Fig. 1.2A by the smoother bottom curve. The choppy top curve depicts only the change in aerodynamic resistance. Fig. 1.2B gives the autocorrelation function of fluctuations of resistance, and Fig. 1.2C shows the same autocorrelation function after removing the constituent having the 27-day period of fluctuation from it. We can see from the graph that the fluctuation correlation interval for the rate of change in the period of revolution is 2 days in the given case.

An increase in the duration of the correlation interval of the second satellite is quite natural, since it was in a higher orbit and owing to the greater rarefaction of the atmosphere, the absolute quantity of the force of resistance arising in its motion was less.

Thus, available test data attest to the fact that for artificial satellites whose trajectories are 350 to 630 km distances from the Earth, the correlation interval of fluctuation in atmospheric resistance extends from a quantity somewhat less than one day to a quantity equal to two days.

Unfortunately, in terms of existing data it is impossible to form an idea on variations in the magnitude of the correlation interval as a function of solar activity and other factors. In article [30] it is noted that they describe the upper boundary of values of correlation intervals. Data on the values of correlation intervals situated near the lower boundary of quantities encountered in practice are still absent from the literature. There are also no data on correlation intervals of fluctuations in aerodynamic resistance at altitutdes less than 350 km, and we may only form a rather approximate idea of them.

The cited experimental data are directly related to the fluctuation rate of the satellite's period of revolution and in some way characterize the magnitude of the correlation interval of fluctuation in the parameters of motion, which are the research goal of this study. It is clear, to begin with, that since the fluctuation rate of the period of revolution is a derivative of this period, we may judge the period correlation interval in terms of the duration of the fluctuation correlation interval and consequently, the semimajor axis and eccentricity of the orbit. In this regard, fluctuation changes of these quantities, at first glance, may be considered stationary processes. This implies that the fluctuation correlation function for the rate of change in the period of revolution of an artificial Earth satellite (AES) is equal to the second derivative of the correlation function of the fluctuations of this period: /27

$$Z_{T'}(\tau) = Z''_{T}(\tau).$$

In turn, as illustrated by the autocorrelation function of exponential form, we can see that the durations of the correlation intervals of the two similar processes are identical. It is useful to note that the exponent, as one possible version of an approximating function, is distinguished by the feature that its second derivative, which must reflect the correlation function of the derivative of an initial random process, reflects the basic properties of the correlation function as does the function per se.

Therefore, we may assert that previously discussed data on the durations of correlation intervals are related not only to fluctuations in orbital acceleration, but also to fluctuations in orbital velocity, period, and the semimajor axis.

As concerns the other orbital elements, random fluctuation in aerodynamic resistance will apparently not have a noticeable effect on them. The only exception is orbital inclination, which will slowly fluctuate under the influence of the force generated by the daily rotation of the atmosphere. A satellite, entrained by the rotating atmosphere, will not only "sense" fluctuations in the density of the medium, but winds as well, whose velocity, according to some data, may reach 320 km/hour. There are still no experimental data, however, which describe the duration of the correlation interval of orbital inclination fluctuations in the known literature. For that reason, we shall consider that the duration of the interval is at least 1-2 days in any case.

It is still impossible to state anything specific about the /28
order of magnitudes which describe the duration of correlation intervals of fluctuation of other orbital elements. We may only suupose that they exceed by many orders the durations of fluctuation correlation intervals of the semimajor axis and orbital inclination.

The general conclusion which ensues from the cited data consists in the fact that the duration of measurement intervals of AES orbital parameters, at altitudes of perigee from 350 to 630 km, should not exceed 1-2 days.

1.3. Signals

Reference and relayed electromagnetic fields within the area of disposition of receiving antenna elements serve as signals in space measuring complexes. These fields may be defined by value sets of intensity at all points in the indicated area.

The field intensity of the reference signals, in particular, may be represented in the following complex form:

$$s_0 = \dot{A}_0(t)\exp(i\omega t), \qquad (1.3.1)$$

where $A_0(t) = \overline{A}_0(t)\exp(i\phi\beta)$ is a complex amplitude; and $\overline{A}_0(t) = A_0(t)\exp[i\phi(t)]$ is a modulating function.

The notations adopted here are: $k = \omega/\upsilon_{ph}$ -- wave number; υ_{ph} - phase velocity of radiowaves; β -- initial signal phase.

The relayed (or reflected) field may be described by the formula

$$s(t) = \dot{A}(t, r)\exp(i\omega t), \qquad (1.3.2)$$

where

$$\dot{A}(t, r) = \overline{A}(t - 2r/v_{gr}) \exp(-i \cdot 2kr) \exp i\beta ;$$
$$\overline{A}(t - 2r/v_{gr}) = A(t - 2r/v_{gr}) \exp [i\varphi(t - 2r/v_{gr})] ;$$

r - instantaneous distance from SV to point of measurement;
v_{gr} -- group rate of propagation.

Let us assume that the source of the relayed field is unique and is integrated with the point of SV position, and the elements of the receiving antennas discretely or continuously fill some limiting area of space which may arbitrarily be called the volumetric antenna area.

The amplitude of the assumed field, as well as the phases of its carrier and modulating oscillations are a function of twice the value of the instantaneous distance between the SV and the point of observation, which is equal to the modulus of difference of two radius-vectors (Fig. 1.3): /29

$$r = |r_S - r_E| \tag{1.3.3}$$

one of which (r_S) describes the instantaneous spatial position of the SV; the second (r_E) -- the point of observation.

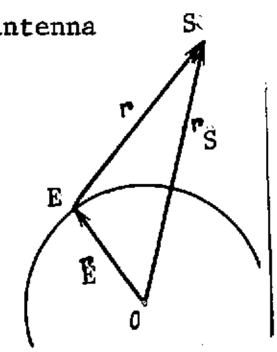

Fig. 1.3. Radius vectors of point of observation r_E, space vehicle r_S, and instantaneous distance between SV and observation point r.

The instantaneous distance from the point of observation to the SV may also be represented in the following manner. Located at some fixed point on the antenna (which may be called its center) is the origin of a topocentric system of coordinates, the radius-vector of the instantaneous point of the antenna is denoted r_A (Fig. 1.4); the instantaneous distance between the SV and the point of observation may also be expressed in the following formula:

$$r = |r_S - r_{Ec} - r_A|. \tag{1.3.4}$$

The source of data on parameters of motion may be not only the field of the reflected or relayed signal. The field generated by the autonomous on-board transmitter may also be such a source; its emission is not synchronized with the emission of ground reference generators (non-feedback operating mode). It seems that in the non-feedback operating mode, the received signal is retarded with respect to the emitted signal by a period of /30

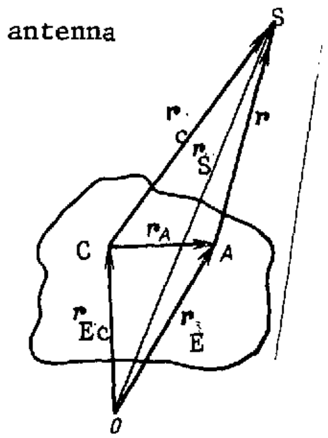

Fig. 1.4. Geometric relationships in measurements using a space complex.

time proportional to the distance between the SV and the point of observation.

The radius-vectors of the SV and the center of the antenna may be given in the form of some regular functions of some number of constant quantities and time. These constants are parameters of SV orbit and the observer's coordinates. The entire set of these constant parameters we will call the parameters of motion. The parameters of motion are usually represented by multidimensional vectors, whose individual constituent parts describe the orbit and the observer's position. We will henceforth denote the vector of the parameters of motion by the letter q. Thus, the instantaneous distance between SV and the observer may be presented as a function of the parameters of motion and time:

$$r = r(q, t).$$

These arguments indicate that amplitude, instantaneous phase, and signal time lag, usually called the signal parameters, in determinant motion are known regular functions of time. For this reason, signals operating in space radiotechnical complexes may be called signals with regularly varying parameters. The regular nature of the relationship of signal parameters as a function of time and the parameters of motion is a specific feature of signals emitted from objects travelling along determinant trajectories.

We may quite arbitrarily divide signal parameters into informative and non-informative. Informative parameters are those which are directly utilized to obtain information on the parameters of motion; non-informative parameters are those which are not directly utilized to obtain similar information.

The informative parameters may be such as instantaneous phase, frequency, time lag, and sometimes even signal amplitude.

Non-informative parameters are either constants, regularly variable, or randomly variable quantities, whose statistical properties are known. The initial signal phase is usually such a parameter. The same information contained in amplitude is often not utilized. /31

Several signal models will be discussed subsequently which differ in non-informative parameter characteristics.

To begin with, we must examine signals with regularly variable amplitude and known initial phase. These type of signals, as we know, are not realized in space measuring complexes, but a signal model having a known initial phase we shall include in those models discussed, since in some instances the properties of actual signals may be profitably compared with signal properties of this hypothetical model. Signals having a known initial phase will be called first model signals.

In addition to signals having a known initial phase, we will also examine isolated signals having an initial phase whose magnitude is constant for the entire existence of the signal and random in transition from one signal realization to another. The amplitude of such signals, which we will call second model signals, vary in accordance with a regular law in correspondence with a change in distance between the SV and the observer. The initial phase is uniformly distributed from 0 to 2π.

The third model corresponds to isolated signals having a random initial phase and amplitude. The law of distribution of the initial phase, as before, is assumed to be uniform, and amplitude conforms to Rayleigh's law of distribution.

It is also expedient to emphasize sequences of signals having random initial phases and amplitudes. Such sequences shall be called fourth model signals. We will ultimately examine continuous signals having slowly fluctuating initial phase and amplitude.

Generally speaking, the electromagnetic field used in space measuring complexes is a complex wave process having a fluctuating phase and amplitude. The fluctuations of parameters of this process are, on one hand, the result of fluctuation effects within the generator (thermal and shot noise, "technical" fluctuations) and on the other hand, the result of random heterogeneity of the medium in which this process is propagated. In this regard, fluctuations generated by various physical factors differ in their statistical properties. Each mechanism has an inherent time and space correl- /32 ation interval, the simple separation of effects due to different mechanisms not always being possible: these processes do not always conform to the principle of superposition. But, taking into account the large duration of the observation interval and the small specific gravity of rapid fluctuations of small intensity, we may be limited to the assumption that signal amplitude and phase fluctuate quite slowly. They remain constant during the correlation interval and fluctuate in conformity to a random law during transition from one correlation interval to another. In relation to experimental data related to modern, high-stability quartz frequency standards used in the AES in conjunction with or without atomic standards, let us assume that the interval of the time correlation of phase fluctuations may reach several seconds and minutes, while the interval of space correlation -- hundreds of thousands and millions of kilometers.

It is apparent, in this regard, that continuous signals having slowly fluctuating amplitude and phase may, at first glance, be represented as a sequence of pulses adjacent to one another, which have random phase and amplitudes. In other words, an analysis of processes in systems having such signals, which we shall call fifth model signals, may reduce to the analysis of processes in systems having fourth model signals.

The received signal in space radiotechnical complexes may thus be represented with the aid of the formula

$$s = s\{\alpha[r(q, t)], \beta, t\}, \qquad (1.3.5)$$

where α -- the vector of regularly variable signal parameters; β -- the vector of signal parameters which are random quantities or random processes; q -- the vector of definable parameters of motion.

In some cases, the parameters of motion must be subclassified as definable and non-definable. For example, in navigation problems for ground objects according to AES, the definable parameters are the parameters of motion of the observer situated on the Earth or near the Earth. The orbital elements are considered given in this regard. In orbital measurements we are given the coordinates of ground points and the orbital parameters are defined. Thus, generally the vector of the parameters of motion should be subdivided into the vector of definable parameters q and the vector of non-definable parameters q_p. The signal at the point of reception is therefore written: /33

$$s = s\{\alpha[r(q, q_p, t)], \beta, t\} . \qquad (1.3.6)$$

We should add that, in general, the signal field is polarized and it must therefore be represented by three components of the corresponding vectors. However to simplify the problem, let us confine ourselves to an examination of only one component of the polarized field, assuming that the type of polarization is taken into account in the design of antennas.

1.4. Brief Characterization of the Field of Random Interference

The influence of diverse natural interferences on the radio channels of space measuring complexes may be reduced to the influence of random vectorial electromagnetic fields on the elementary antennas of the complexes. The antennas, in general, are completely or partially polarized, heterogeneous, anisotropic, and non-stationary. Of the greatest practical interest are the random fields formed as a result of superposition of a great number of fluctuation fields created by sets of more or less uniformly concentrated in space sources of noise emission. These fields conform to the normal law of distribution, represented by comparatively simple analytic relationships, which are extremely suitable for use in conducting diverse studies [2].

The ideas on random electromagnetic fields were formulated as natural generalizations of ideas on random processes which include functions of time, whose instantaneous values are random quantities which conform to specific laws of distribution. However, in identifying a random process with a specific set of random quantities, we must take into account that this set is not equivalent to a simple sum of individual random quantities and represents a much more complicated phenomenon. The particularity of a random process is that between the elements of the set of random quantities into which it may be factored, there may exist a specific interrelationship. For that reason, a random process is characterized by a multidimensional law of probability distribution, which generally is not divided into parts related to separate /34
random quantities, and breaks down into a large number of one-dimensional laws only if there is no relationship between its instantaneous values. Moreover, random interference with which we must deal in radio technology is continuous in nature and, strictly speaking, is identical to an infinite set of random quantities. For this reason, the distributive law of interference is represented not by a function, but by a functional of the probability density [20].

The random electromagnetic field is a set of vectorial random processes effective in some area of space or, in other words, a vectorial random process, whose instantaneous value is not just a function of time, but also a function of the spatial coordinates of the point of observation. It is identical to three scalar random fields, each of which is described by the corresponding functional of the probability density. In this regard, the separation of the distributive law of random field realizations into distributive laws of random processes at individual points in space is likewise impossible. This division is only possible in the absence of a correlative relationship between the corresponding random processes.

Analytic expressions for the functionals of the probability density of scalar components of the random vectorial field of the normal type may be derived from the expression for the function of distribution of discrete values of a normal random process, which has the following form:

$$w(\mathbf{n}_t) = \frac{1}{(\sqrt{2\pi})^k \sqrt{\det \mathbf{B}_n}} \exp\left\{-\frac{1}{2}[\mathbf{n}^T \mathbf{B}_n^{-1} \mathbf{n}]\right\}, \tag{1.4.1}$$

where n -- is a k-dimensional vector-column, whose components are elements of random process selection, whose volume is equal to k; n^T -- the transposed vector-column; B -- the correlation matrix of interference, which is of square form k x k; det B_n -- the matrix determinant.

By introducing the matrix C, the inverse to the correlation matrix B_n, the sign of the exponent of formula (1.4.1) may be written in the form

$$-\frac{1}{2}\mathbf{n}^T \mathbf{B}_n^{-1} \mathbf{n} = -\frac{1}{2}\mathbf{n}^T \mathbf{C}\mathbf{n} = -\frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{k} n_i n_j C_{ij}, \tag{1.4.2}$$

where, by definition, $CB_n = 1$ -- a unit matrix which is equivalent to the relationship

$$\sum_{m=1}^{k} c_{mj} b_{im} = \delta_{ij}; \quad \delta_{ij} = \begin{cases} 1 & \text{where } i = j, \\ 0 & \text{where } i \neq j. \end{cases}$$

Let us compose an expression for the functional of the probability density of a normal random process. The unknown functional is derived from (1.4.1), if the number of divisions of the segment of time of existence of noise is to approach infinity (and thus, if the time interval between divisions approaches zero): /35

$$w[\mathbf{n}(t)] = \lim_{\substack{k \to \infty \\ \Delta \to 0}} w[\mathbf{n}_t]. \tag{1.4.3}$$

With an increase in the number of divisions of the area of fluctuation of the argument, the double summation (1.4.2) in the exponent sign of (1.4.1) approaches the double integral

$$\lim_{\substack{k\to\infty \\ \Delta\to 0}}\left\{-\frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{k} n_i n_j c_{ij}\right\} = -\frac{1}{2}\int_0^T\int_0^T n(t_1)\,n(t_2)\,a(t_1,\ t_2)dt_1\,dt_2, \qquad (1.4.4)$$

where Δ is the distance, approaching zero, between two values of the argument. This equality is valid if in the limiting process $\Delta \to 0$ the relationship

$${}_{j} = a(t_i,\ t_j)\,\Delta^2, \qquad (1.4.5)$$

is satisfied; it may be written as

$$d^2c = a\,dt_1 dt_2$$

or as

$$a = d^2c/dt_1 dt_2,$$

where c_{ij} -- an element in matrix C, inverse to the correlation matrix. In other words, if we differentiate the random process n(t), the infinite double summation may be written as a double Stieltjes integral

$$\lim_{k\to\infty}\left(-\frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{k} n_i n_j c_{ij}\right) = -\frac{1}{2}\int_0^T\int_0^T n(t_1)n(t_2)d^2c(t_1,\ t_2) = -\frac{1}{2}\int_0^T\int_0^T n(t_1)n(t_2)\,a\,dt_1\,dt_2, \qquad (1.4.6)$$

the function $a(t_1,\ t_2)$ being connected to the correlation coef- /36
ficient by the integral equation

$$\int_0^T B(t_1,\ t_2)\,a(t_1,\ t_2)\,dt = \delta(t_1 - t_2), \qquad (1.4.7)$$

which is an analog of the equation

$$\boldsymbol{CB} = \boldsymbol{I}. \qquad (1.4.8)$$

After introducing the functional

$$\int_0^T n(t_2)\,a(t_1,\ t_2)dt_2 = z(t), \qquad (1.4.9)$$

the limit of quadratic form in the exponent of the normal distributive law may be rewritten as follows:

$$\lim_{n\to\infty}\left(-\frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{k}n_i n_j c_{ij}\right)=-\frac{1}{2}\int_0^T n(t)z(t)dt, \tag{1.4.10}$$

where z(t) is defined by equation (1.4.9), which is equivalent to the condition

$$n(t)=\int_0^T B(t,\ u)\,z(u)du\,, \tag{1.4.11}$$

which is easy to verify by multiplying equation (1.4.7) by $n(t_2)$ and integrating both parts from 0 to T.

When a number of divisions of the range of existence of arguments approaches infinity, the coefficient before the exponent also approaches infinity, but this does not cause any difficulties since in the problems under discussion we are using a ratio of functionals of probability densities which remains finite.

The expression for the functional of the probability density of a stationary uncorrelated random process ("white" noise) is somewhat simplified. Since in this regard

$$B(t_1,\ t_2)=\frac{N_0}{2}\,\delta(t_1-t_2), \tag{1.4.12}$$

where N_0 -- the spectral density of noise, then z(t) = n(t) and

$$\lim_{n\to\infty}\left(-\frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{k}n_i n_j c_{ij}\right)=-\frac{1}{N_0}\int_0^T n^2(t)dt. \tag{1.4.13}$$

Therefore, for uncorrelated noises, the integral in the exponential sign expresses the energy of the fluctuation process. /37

As was noted, a random process is a function of a point in a four-dimensional time-space manifold. This means that the probability density of a discrete sample of values of the normal field is represented by a formula analogous to formula (1.4.1):

$$w\,[n_{t,r}]=K\exp\left\{-\frac{1}{2}\,[\mathbf{n}^T\,\mathbf{B}_n^{-1}\,\mathbf{n}]\right\}, \tag{1.4.14}$$

but the volume of the sample now increases substantially. The sample is formed, in this case, not only as a result of the division of effect time of interference on segments of Δt, but also as a result of division of the spatial region of the field setting into several elementary volumes of ΔV. If the number of such elementary volumes is equal to ℓ, the volume of the sample will constitute $m = k\ell$. If the volume sample approaches infinity, we will switch from the distributive function of (1.4.14) to the functional distributive density of a large number of random field realizations. This limiting process is similar to the one given in formula (1.4.4), but now in place of the function $n(t)dt$, we must use the function $n(t, r)\,dtdV$ and integrate not only in terms of time, but also in terms of the volume in which the receiving antennas are arranged. It is likewise clear that instead of the time correlation coefficient $a(t_1, t_2)$ we must use the coefficient of space-time correlation $a(t_1, t_2; r_1, r_2)$. As a result, we arrive at the functional

$$-\frac{1}{2}\int_0^T\int_V n(t, \mathbf{r})\, z(t, \mathbf{r})\, dt\, dV, \tag{1.4.15}$$

whose subintegral coefficient $z(t,r)$ satisfies the Fredholm equation

$$n(t, \mathbf{r}) = \int_0^T\int_V B(t, \mathbf{r}, \tau, \rho)\, z(t, \rho)\, dt d V. \tag{1.4.16}$$

Ultimately, for the functional of the probability density of a normal random electromagnetic field we derive the relationship

$$w[n_{t,r}] = K\exp\left\{-\frac{1}{2}\int_0^T\int_V n(t, \mathbf{r})\, z(t, \mathbf{r})\, dt dV,\right. \tag{1.4.17}$$

which is extremely general in nature and is suited for describing /38
both homogeneous stationary and isotropic, as well as heterogeneous, nonstationary, and anisotropic random fields.

It should be mentioned, however, that natural random electromagnetic fields, especially fields of thermal noise, may at first glance be considered as stationary homogeneous and isotropic fields. In addition, the width of the fluctuation spectrum usually exceeds the width of the signal spectrum, making it possible to approximate the spectrum of actual interference, which is a function of frequency as interference having a uniform spectrum ("white" noise), i.e., interference whose time correlation coefficient is represented by a Dirac delta function. Finally, the dimension of the space correlation interval of thermal noises does not exceed the magnitude of wave length order, allowing us to approximate the function of the space correlation by a Dirac delta function as well. Therefore, with the foregoing assumptions

$$B(t, \mathbf{r}, \tau, \rho) = \frac{N_0}{2}\delta(t-\tau)\delta(\mathbf{r}-\rho), \qquad z(t, \mathbf{r}) = \frac{2}{N_0}n(t, \mathbf{r}). \tag{1.4.18}$$

Consequently, the functional of the probability density of a homogeneous stationary delta-correlated random field is represented by the formula

$$w[n_t, \mathbf{r}] = K\exp\left\{-\frac{1}{N_0}\int_V\int_0^T n^2(t, \mathbf{r})\,dt\,dV\right\}, \tag{1.4.19}$$

where N_0 -- specific spectral density of fluctuations, which is equal to the energy scattered in an isolated volume per unit time.

Chapter 2 /39

EVALUATION OF PARAMETERS OF MOTION AND THE OPTIMUM FILTRATION OF SIGNALS HAVING REGULARLY VARIABLE PARAMETERS

2.1. Methods of Direct Evaluation of Parameters of Motion in Terms of the Field of the Received Signal

The research methods may be characterized in the following manner.

To signal s, carrying information about the motion of a SV or terrestrial (near-Earth) observer are additively superimposed fluctuation interferences n. At the reception point, the effective summary signal is

$$y = s + n. \tag{2.1.1}$$

Here s = s(t, r), n = n(t, r), y = y(t, r) -- functions of coordinates and time which may be considered multidimensional vectors, whose components are expansion terms of these functions into a series in conformity to Kotel'nikov's theorem.

As was noted earlier, the signal is a determinant or quasi-determinant electromagnetic field having random and regularly variable parameters, resulting from complex nonlinear relationships having specific parameters of motion:

$$s = s\{\alpha[r(q,\ q_p,\ t)],\ \beta,\ t\}\ . \tag{2.1.2}$$

Interference which distorts the signal is a random, stationary electromagnetic field.

The problem is, with respect to an additive mixture of noise and signal in a given area of space, to define the magnitude of the vector of definable parameters q.

The primary distinctive feature of this problem is that it is not the parameters of the signal which function as directly definable quantities, but the parameters of motion, i.e., geometric and kinematic quantities which describe the spatial position and motion of space or terrestrial object. As we know, the theory of evaluations in radar technology is usually applied in evaluating /40
signal parameters; the process of defining the parameters of motion extends beyond the framework of the evaluative process and is seen as a problem of secondary signal processing (information).

Another substantial feature of the problem is that signals and noise are examined for an interval of time in which there is significant fluctuation in the juxtaposition of emission source and receiver. Moreover, a particularity of research is the use of receiving systems which not only consist of discrete point antennas, but also of a rather large number of elements which are discretely or continuously filling a specific area of space.

Under these conditions, it would appear that it is objectively possible to directly define the entire set of parameters which describe the spatial position and motion of a SV or ground observer.

As we can see from the formulation of the problem, it is statistical in nature and may be reduced to evaluating the magnitude of the parameters of the resulting distributive law of received signals, taken as a multidimensional random quantitiy. The received signal $\underline{y}$ is indeed a known function of several random vectors q, q_p, β, n, whose distributive laws are known. Consequently, we may compute the resulting distributive law of the vector of $\underline{y}$, as well as the conventional distributive laws of the type $w\ (y/q)$, in terms of which we may find the aposteriori distributive law of probabilities $w\ (q/y)$.

Having this law at our disposal, we may make a specific evaluation of the parameter q. Obviously, the most preferable are optimum evaluations, as which we customarily understand those evaluations which ensure minimizing of the mean risk (or mean losses) in defining error cost, i.e., evaluations which satisfy the condition

$$\Pi = \iint \Pi(q, \hat{q})\, w(q, \hat{q})\, dq d\hat{q} = \min, \tag{2.1.3}$$

where q and $\hat{q}$ -- vectors of definable parameters of motion and its evaluation; $\Pi(q, \hat{q})$ -- cost of errors (function of losses); $w\ (q, q)$ -- combined probability density of quantities q and $\hat{q}$.

We shall limit ourselves to an examination of optimum evaluations of the Bayes type, since in processing metric data in most all-purpose and specialized space radar complexes there is more or less accurate apriori information on the parameters of motion. The exception are detection complexes, which we will not discuss at this point. /41

We know that regardless of the sampling of error cost, the evaluative problem comes down to the definition of the aposteriori probability density of the known parameter. In defining the coordinates of the center of gravity of an aposteriori distribution, in particular, we obtain an optimum evaluation corresponding to the loss function squared

$$\Pi(q, \hat{q}) = (q - \hat{q})^2.$$

The coordinates of the aposteriori probability density maximum correspond to the loss function of the form

$$\Pi(q, \hat{q}) = 1 - \delta(q, \hat{q}),$$

where δ -- the delta function.

In turn, the aposteriori probability density of the unknown vector of the parameters may be represented as a product of the probability density of apriori data errors w(q) and the ratio probability of the selection of the received signal and noise mixture $\ell(y/q)$:

$$w(y/q) = K w(q) l(y/q), \tag{2.1.4}$$

where, as we know, the ratio of sampling probability densities is recorded in the presence of a signal and in its absence. Taking into account the presence of undefinable parameters of motion and non-informative signal parameters, we may write the following expression for the ratio of probability:

$$l(y/q) = \frac{\iint\limits_{-\infty}^{\infty} w(y, q, q_n, \beta) w(q_n) w(\beta)\, dq_n d\beta}{w(n)}. \tag{2.1.5}$$

The constant coefficient K in formula (2.1.4) serves to normalize the aposteriori distributive density. It is equal to

$$K = \frac{w(n)}{\int\limits_{-\infty}^{\infty} w(q) l(y/q)\, dq}. \tag{2.1.6}$$

Beneath the integral sign in the ratio of probability appears the conventional probability distribution density of sample selection y, i.e., the distribution density of a sampling with several fixed values of the parameters q, q_p, and β. It appears that with fixed values of these parameters, the probability distribution /42 density of the sampling will coincide with the probability density of interference and may be expressed by the equation

$$w(y/q,\ q_n,\ \beta) = w_n[y - s(q,\ q_n,\ \beta)], \tag{2.1.7}$$

where $w_n(n)$ -- the probability distribution density of interference.

In turn, the field realization probability density which appears in the denominator of the formula for the ratio of probability, in the absence of a signal, is expressed as

$$w(n) = w_n(y). \tag{2.1.8}$$

Ultimately, the formula for the ratio of probability acquires the following form:

$$l(y/q) = \frac{\int\int_{-\infty}^{\infty} w_n[y - s(q,\ q_n,\ \beta)]\, w(q_n)\, w(\beta)\, dq_n\, d\beta}{w_n(y)}. \tag{2.1.9}$$

If the vector of the parameters of motion does not contain known undefinable parameters q_p, this expression is simplified and acquires the form

$$l(y/q) = \frac{\int_{-\infty}^{\infty} w_n[y - s(q,\ \beta)]\, w(\beta)\, d\beta}{w_n(y)}. \tag{2.1.10}$$

Therefore, having at our disposal analytic expressions for the signal and for the function or functional of the probability density of interference, we may define the probability density of the sampling or realization of the field of signal-and-interference: in the presence of apriori data on the parameters of motion, this lets us derive the conventional distribution density of the vector of values of unknown parameters of motion and compose an optimum evaluation of them.

In summarizing, we can observe that we have essentially reduced the task of defining the parameters of motion in space measuring complexes to the generalized task of filtering radio signals.

In this problem, the measuring means of the complexes are seen as a unified space-time filter, functioning in the interval of /43 constancy of definable parameters of motion and shaping the evaluation of the magnitudes of these parameters which is optimum from the standpoint of the defined criteria.

A feature of generalized space-time filters, among which may be related space, rocket, and other radio-technical complexes, is the complex nature of the relationship between definable parameters of motion and field parameters of the signal, which may be nonlinear and variable in time.

2.2. Ratio of Probability

The ratio of probability is the most substantial element of the probability density of a received mixture of signal and noise. The formula relationships which describe the ratio of probability for different signal models may be derived by substituting in formula (2.1.9) analytic expressions for signals and probability densities of interference, taking account of the distributive law of vectors of non-informative parameters β and undefinable parameters of motion, q_p. To simplify the problem, we will limit ourselves to the case where there are no unknown undefinable parameters, some parameters are known, and all unknowns enter into the category of definable quantities. In this respect, we will calculate ratios of probability only for an isolated signal having random initial phase, uniformly distributed in the interval from 0 to 2π : $\omega(\beta) = 1/2\pi$.

Assuming that only initial phase β is related to the number of non-informative parameters, from formula (2.1.10) for the ratio of probability we yield the following equation:

$$l(y/\mathbf{q}) = e^{-\frac{E}{N_0}} \frac{1}{2\pi} \int_0^{2\pi} \exp\left[\frac{2}{N_0} \int_V \int_T y(t, r)\, s(t, r, \beta)\, dV dt\right] d\beta, \qquad (2.2.1)$$

where

$$E = \int_V \int_T s^2(t, r, \beta)\, dt dV \qquad (2.2.2)$$

-- signal energy effective in volume V.

The ratio of probability is expressed with the aid of the integral from the exponential function, whose argument is the product of some constant quantity X the space-time integral taken in terms of four-dimensionsal volume /44

$$z=\iint_{VT} y(t, r)s(t, r, \beta)dVdt. \qquad (2.2.3)$$

In carrying out calculations, we will be using a complex form for writing the signal, interference, and the mixture of signal and interference:

$$s(t, r)=\dot{A}(t, r)\exp[i(\omega t+\beta)], \qquad (2.2.4)$$

$$n(t, r)=\dot{N}(t, r)\exp(i\omega t), \qquad (2.2.5)$$

$$y(t, r)=\dot{Y}(t, r)\exp(i\omega t), \qquad (2.2.6)$$

where $\dot{A}(t,r) = \bar{A}\exp(-ikr) = A(t, r)\exp i\phi(t, r)\exp(-ikr)$; $\dot{N}(t, r) = N(t,r)\exp[i\phi_i(t, r)]$; $\dot{Y}(t, r) = \dot{A}(t, r)\exp i\beta + N(t,r)$.

The use of complex expressions permits us to give integral (2.2.3) the following form:

$$z=\iint_{TV} y(t, r)s(t, r)dV dt=\frac{1}{2}\mathrm{Re}\left\{\iint_{TV}\dot{Y}(t, r)\exp(i\omega t)\dot{A}(t, r)\times\right.$$
$$\times\exp[i(\omega t+\beta)]dVdt+\iint_{TV}\dot{Y}(t, r)\exp(i\omega t)\dot{A}^*(t, r)\times$$
$$\left.\times\exp[-i(\omega t+\beta)]dVdt\right\}. \qquad (2.2.7)$$

Complex amplitudes A and Y fluctuate within a four-dimensional volume TV rather slowly, and the duration of observations greatly exceeds the magnitude of the period of high-frequency oscillation; therefore, below the sign of the time integral of the first component of summation (2.2.7) will be found a rapid-oscillation function approaching zero. Consequently,

$$z=\frac{1}{2}\mathrm{Re}\iint_{TV}\dot{Y}\dot{A}^*\exp(-i\beta)dVdt. \qquad (2.2.8)$$

By introducing the notations /45

$$Z = \frac{1}{2}\left|\int_T\int_V \dot{Y}(t,\ r)\dot{A}^*(t,\ r)\,dV dt\right|, \tag{2.2.9}$$

$$\cos\theta = \mathrm{Re}\int_T\int_V \dot{Y}(t,\ r)\dot{A}^*(t,\ r)\,dV dt/Z, \tag{2.2.10}$$

$$\sin\theta = \mathrm{Im}\int_T\int_V \dot{Y}(t,r)\dot{A}^*(t,\ r)\,dV dt/Z, \tag{2.2.11}$$

the integral of (2.2.3) may be given the following form: $z = Z\cos(\beta - \theta)$.

We ultimately derive the following relationship for the ratio of probability:

$$l(y/q) = \frac{1}{2\pi} e^{-\frac{E}{N_0}} \int_0^{2\pi} \exp\left[\frac{2Z}{N_0}\cos(\beta - \theta)\right] d\beta = e^{-\frac{E}{N_0}} I_0\left(\frac{2Z}{N_0}\right), \tag{2.2.12}$$

where $I_0(x)$ -- a modified zero-order Bessel function. The integral

$$Z = \frac{1}{2}\left|\int_T\int_V \dot{Y}(t,r)\dot{A}^*(t,\ r)\,dV dt\right| \tag{2.2.13}$$

is called the space-time correlation integral or the intercorrelation function of the signal field and the field of the received mixture of signal and noise.

Therefore, the procedure for deriving the optimum evaluation of parameters of motion reduces to a definition of the space-time correlation integral Z, and optimum filtration of the signal includes transmitting the received signal and noise mixture through a set of correlation devices; as a reference signal, signals free from interference are fed in (these signals are effective in the corresponding points in space); and the summing of output effects of all correlation devices located within the region V. In practice, however, we can not have at our disposal signals free of interference at the points of reception, because the formation of such signals would require the disposition of precise values for the parameters of motion, whose definition is the end purpose of the measurements. As a consequence, there is a lack of precise apriori information about the parameters of motion on the receiving end, and only this information may actually be utilized to form the reference signal. Consequently, the practical meaning may only gained by using reference signals formed on the basis of apriori data and only with some degree of accuracy corresponding to the actually effective signal. Therefore, the space-time correlation integral will imply /46

$$Z=\frac{1}{2}\left|\int\limits_{T}\int\limits_{V}\dot{Y}(t,\ r)\dot{A}_a^*(t,\ r_a)\,dV\,dt\right|, \tag{2.2.14}$$

in which appears a reference signal $A_a(t,\ r_a)$, formed in terms of apriori data about the parameters of motion.

Integral (2.2.14) is similar in form to the correlation integral which describes the procedure of optimum filtration of radar signals [27]. Between these integrals, however, there is a very great distinction. The primary distinction is due to the features of the received signal and includes the fact that if a signal having constant parameters is used as the reference signal, in this case we would use a signal having regularly variable parameters, formed at reception points in terms of apriori data as the reference signal.

Another distinctive feature of the space-time correlation integral is that signal field strength does not appear in it, but the signal's volumetric density does.

Thus, the definition of the ratio of probability for the field of an isolated signal having a constant random phase (in terms of the earlier given classification -- a second model signal) reduces to the definition of the correlation integral equal to the modulus of a quadruple integral from the product of the densities of reference signal and received signal and noise mixture.

It follows from this analysis that for an isolated signal having a zero initial phase (i.e., for a first model signal), the ratio of probability is expressed by the formula

$$l(y/q)=\exp(-E/N_0)\exp(2Z_1/N_0), \tag{2.2.15}$$

where

$$Z_1 = \frac{1}{2} \operatorname{Re} \int_T \int_V \dot{Y}(t, r) \dot{A}_a^*(t, r_a) dV dt. \tag{2.2.16}$$

These relationships are derived from relationships (2.2.1) and (2.2.8), if an initial signal phase β equal to zero is placed in them. /47

Let us now cite expressions for the ratio of probability of third and fourth model signals. From the formal standpoint, the deduction of these relationships coincides with the deduction of formulas for the corresponding models of radar signals [27], and thus there is no need to derive it.

For isolated signals having random phase and amplitude, the ratio of probability has the form

$$l(y/q) = \frac{N_0}{E + N_0} \exp\left(\frac{1}{N_0} \frac{Z^2}{E + N_0}\right). \tag{2.2.17}$$

We have to note that here the initial signal phase is assumed to be uniformly distributed from 0 to 2π, amplitude fluctuates in conformity to some regular law, and its maximum value is random and is distributed in conformity to the Rayleigh law; Z and E are given by formulas (2.2.14) and (2.2.2), respectively.

Finally, for a signal having independently fluctuating amplitude and phase (fourth model signal), the ratio of probability is expressed by the formula

$$l(y/q) = \prod_k \frac{N_0}{E_k + N_0} \exp \frac{1}{N_0} \frac{Z_k^2}{E_k + N_0}, \tag{2.2.18}$$

where

$$Z_k = \frac{1}{2} \left| \int_{T_k} \int_V \dot{Y}(t, r) \dot{A}_a^*(t, r_a) \, dV dt \right|;$$

$$E_k = \int_{T_k} \int_V s^2(t, r) \, dV dt = \frac{1}{2} \int_{T_k} \int_V \dot{A} \dot{A}^* \, dV dt;$$

T_k -- the duration of the correlation interval of fluctuations in amplitude and phase; k -- the number of correlation intervals falling within the limits of the measurement interval.

Generally speaking, integration in terms of volume should be done within the area of spatial correlation of fluctuations in amplitude and phase, but we have done it in terms of the entire area in which the elements of the receiving antennas of the space radar complex V are located, because the dimensions of the area of space correlation usually exceed the dimensions of the area of arrangement of the complex's antennas.

In summation, we may note that the definition of the ratio of probability reduces to the definition of the space-time correlation integral of the form (2.2.14) or (2.2.16). /48

The correlation integral of any form may be written as a sum of two components:

$$Z = | Z_s + Z_n |, \tag{2.2.19}$$

one of which describes the result of interaction of the received and reference signals, the other -- the result of interaction of the reference signal and interference. The first component

$$Z_s = \frac{1}{2} \int_T \int_V \dot{A} \dot{A}_a^* \, dV dt$$

is called the autocorrelation function of the signal (AFS); the second component

$$Z_n = \frac{1}{2} \int_T \int_V \dot{N} \dot{A}_a^* \, dV dt$$

-- the intercorrelation function of the reference signal and interference. With a strong signal, the second component is small vis-a-vis the first. Therefore,

$$Z \approx | Z_s | = \frac{1}{2} \left| \int_T \int_V \dot{A}(t, r) \dot{A}_a^*(t, r_a) \, dV dt \right|, \tag{2.2.20}$$

and the properties of the correlation integral may be judged in terms of the properties of the autocorrelation function of the signal of (2.2.20).

2.3. Properties of the Autocorrelation Function of a Signal Having Regularly Variable Parameters

The autocorrelation function of a signal having regularly variable parameters does not differ substantially from an autocorrelation function of a signal having constant parameters. We shall enumerate the properties which a common for autocorrelation functions of both types.

1. The autocorrelation function is a function of apriori values of the parameters of motion.

In expanding the expression for the instantaneous distance between the point of observation and the SV, we yield from formula (2.2.20) the following /49

$$Z(\mathbf{q}_a)=\frac{1}{2}\left|\int\limits_T\int\limits_V \bar{A}\left[t-\frac{2}{v_{gr}}\left|\mathbf{r}_s(\mathbf{q}_s,\ t)-\mathbf{r}_e(\mathbf{q}_e,\ t)\right|\right]\exp\left[-2ik\left|\mathbf{r}_s(\mathbf{q}_s,\ t)-\mathbf{r}_e(\mathbf{q}_e,\ t)\right|\right]\bar{A}_a^*\left[t-\frac{2}{v_{gr}}\left|\mathbf{r}_s(\mathbf{q}_{sa},t)-\mathbf{r}_e(\mathbf{q}_{ea},\ t)\right|\right]\exp\left[2ik\left|\mathbf{r}_s(\mathbf{q}_{sa},\ t)-\mathbf{r}_e(\mathbf{q}_{ea},\ t)\right|\right]dVdt\right|. \quad (2.3.1)$$

This formula is an analytic expression of the functional relationship between values of AFS and apriori values of the parameters of motion q_{sa}, q_{ea}. The relationship between these quantities is more complex in nature than the relationship between AFS values of a signal having constant parameters and the parameters of the reference signal. Below the integral sign stands the product of two time functions, whose parameters (amplitude and phase) are variable and fluctuate with the passage of time. The first is the signal received by the observe (the observer's position is described by the vectorial quantity q_e) from the SV, whose parameters of motion are q_s (all parameters q_s and q_e or some of them are unknown parameters of motion). Second, or reference, signal is shaped at the receiving point in terms of apriori data on the parameters of SV motion. Its parameters fluctuate with time in accordance with the law of fluctuation of distance from observer to SV which corresponds to apriori data on the orbit and position of the observer.

Formula (2.3.1) for the autocorrelation function of a signal having regularly variable parameters may be written in a somewhat modified form. Using the notations

$$r_s(q_s, t) - r_e(q_e, t) = r,$$

$$r_e(q_{sa}, t) - r_e(q_{ea}, t) = r_a,$$

$$r = r_a + \Delta r, \quad t - 2r/v_{gr} = t'$$

and dropping a line in the integration variable, we derive /50

$$Z(q_a) = \frac{1}{2}\left| \int\limits_V \int\limits_T \overline{A}(t)\,\overline{A}_a^*\left(t + \frac{2\Delta r}{v_{gr}}\right) \exp(-i2k\Delta r)\,dV dt \right|. \qquad (2.3.2)$$

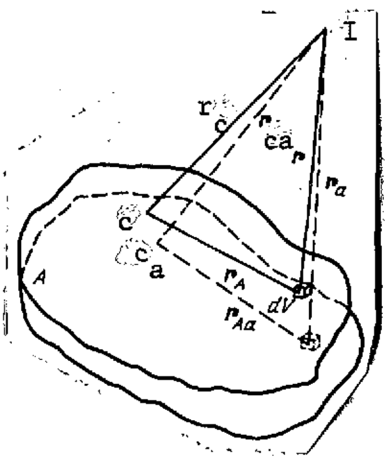

Fig. 2.1. Relations between geometric quantities used in defining the correlation integral.

Henceforth, the limits of integration will be denoted as before by the symbols VT, bearing in mind that integration is implemented within the limits of the space-time area of existence of the received signal. The spatial position of this area is defined by the position of the receiving antennas; the time position -- corresponds to the moments of formation and termination of signal activity at points of location corresponding to elements of the antenna systems.

In preceeding formulas for autocorrelation functions, integration was carried out with respect to time and space within the area occupied by the receiving antenna, while r implied the instantaneous distance of the receiving antenna volume from the emitter. If we adopt some point C, removed from the source by the distance r_c as the origin of the system of coordinates in which the area of integration is given (Fig. 2.1), we may adopt as the integration variable the quantity r_A, which is associated with the quantities r_c and r by the equation

$$r = r_c - r_a.$$

In the expression for the complex amplitude of the reference signal there appears the instantaneous distance of the element of an imaginary receiving antenna volume, located at a point having apriori coordinate values. If this point is designated with the letter c_a, and the corresponding vector is given the index a, the apriori value of the instantaneous range in the expression for the complex amplitude of the reference signal may be written as /51

$$r_a = r_{ca} - r_{Aa}.$$

Therefore, the expression for the autocorrelation function of the signal may be brought to the form

$$Z=\frac{1}{2}\left|\int_T\int_V \overline{A}\left[t-\frac{2\,|\mathbf{r}_c-\mathbf{r}_A|}{v_{gr}}\right]\times \right.$$
$$\left.\times \overline{A}_a^*\left[t-\frac{2\,|\mathbf{r}_{ca}-\mathbf{r}_{Aa}|}{v_{gr}}\right]\exp(-2ik\Delta r)\,dV\,dt\right|, \quad (2.3.3)$$

where A(x) = A(x) exp [i (x)]; r -- the different in ranges from source to elementary volumes, one of whose coordinates corresponds to the actual position of some elementary volume of the receiving antenna; the coordinates of the other -- to the position which this volume would occupy if the antenna were placed in space in conformity to apriori data, i.e., if the antenna center were placed where it is situated apriori, and the "body" of the antenna were turns about three mutually perpendicular axes at several angles -- likewise in conformity to apriori information. In the general case, differences in range between the mentioned elementary volumes may fluctuate in the course of time due to the motion of the source. The remaining notations are given by the formulas

$$r_c = r_{ce}(q_e) - r_s(q_s),$$

$$r_{ca} = r_{cea}(q_{ea}) - r_{sa}(q_{sa}).$$

The letters q_e and q_s, as before, denote the parameters of motion of the observe and the space vehicle, and the vectors r_e and r_s describe the instantaneous position of the observer and SV.

The integration in formula (2.3.2) is done with respect to a four-dimensional space-time area, within the limits of which there appears beneath the integral sign the product of electro-

magnetic fields which differs from zero. One of these fields -- the field established by all elements of the antenna devices of the space complex. Because the evaluation of the parameters of motion, in terms of the problem's meaning, is not done over the /52
entire field generated by the source but only for that part of it which is perceived by the receiving antennas, the amplitude of the unused electromagnetic field is considered to be zero at all points situated outside the receiving antennas.

The second factor of the subintegral expression is the large number of reference signals shaped at the reception point, whose parameters generally differ from the parameters of the received signals due to the distinction in apriori and actual values of SV coordinates and those of the reception point; as well as due to time measurement errors due to the misalignment of time scales at the emission and reception points.

In discussing one-dimensional problems of "time" filtration of signals, the location of the assigned field of the reference signal on the time scale, as we know, is reflected with a shift with respect to the area of existence of the received signal. In the case of space-time filtration, with a strictly formal approach, we should take into account not only the time, but also the space shift in the area of assignment of the large number of reference signals. In a physically realized filter, however, the assigned area of the large number of reference signals coincides with the assigned area of the large number of received signals, because the reference signals should be shaped at all reception points. Consequently, the space assignment areas of received and reference signals are identical and their position coincides with the position of the area of location of the elements of the antenna devices. Therefore, the are of space, within whose limits integration is done, coincides with the area of location of elements of the antenna devices. Integration with respect to time will likewise be done within the limits of the existence time of the received signal.

2. The autocorrelation function is a function of definable corrections for apriori values of the parameters of motion.

With a small difference between the apriori and actual data, i.e., with a fairly high accuracy of apriori data, it is possible to use a linear approximation of instantaneous range to the SV:

$$r(q_s, q_e, t) = r_a(q_{sa}, q_{ea}, t) + \sum_{i=1}^{m} \frac{\partial r}{\partial q_i} \Delta q_i,$$

where $q = \{\Delta q_{ei}, \ldots, \Delta q_{ei}, \Delta q_{s(i+1)}, \ldots, \Delta q_{sm}\}$ -- the dif- /53 ference between actual and apriori values of the parameters of motion, which will likewise be called corrections for apriori values of these parameters, and the derivatives dr/dq_1 taken at points corresponding to their apriori values.

Allowing for the latter relationship, the autocorrelation function of (2.3.2) may be written as

$$Z(\Delta \mathbf{q}) = \frac{1}{2}\left|\int_V\int_T \overline{A}_a^*\left(t+\frac{2}{v}\sum_{l=1}^{m}\frac{\partial r}{\partial q_l}\Delta q_l\right)\times \overline{A}(t)\exp\left(-i2k\sum_{l=1}^{m}\frac{\partial r}{\partial q_l}\Delta q_l\right)dV dt\right|. \tag{2.3.4}$$

Therefore, the ACF of the signal having regularly variable parameters is a function of the vector of corrections for the parameters of motion, whose dimension is defined by the nature of the problem in question.

3. The maximum value of the autocorrelation function is equal to the signal energy detected in the limits of volume V. It is attained with the complete coincidence of apriori values of the parameters of motion with their actual values. In proportion to the divergence of apriori and actual values of the parameters of motion, the magnitude of the ACF decreases.

It appears that the more accurate the apriori information is, the less difference there is between the received and reference signal and the greater the output effect of the space-time filter approaches the magnitude

$$Z_{\max} = \frac{1}{2}\left|\int_V\int_T \overline{A}(t,\ r)\,\overline{A}^*(t,\ r)\,dV dt\right|,$$

equal to the sum energy of the signal perceived within the volume occupied by all elements of the antenna systems of the complex. Inversely, the greater the difference between the received and reference signals, the less the correlation integral. Therefore, by the magnitude of the output effect of the space-time filter we may judge the degree of distinction between apriori and actual values of the parameters of motion, i.e., the magnitude of the actual values of these parameters.

By means of varying the apriori values of the parameters of motion, it is possible to discover the value of these parameters at which the autocorrelation function of the signal and thus, the correlation integral too achieve their maximum value. It appears that the maximum ACF will be attained at values of apriori data equal to the actual values of the parameters of motion or, in any event, at values comparatively close to them. /54

Therefore, the optimum procedure for defining the parameters of motion may be given in the following manner. To discover the optimum evaluation of the parameters of motion, we must compute the correlation integral (2.2.13), using existing apriori values of the unknown parameters. Then, by varying the apriori values of the parameters of motion, we must seek the values of apriori data which ensure maximizing of the correlation integral. Values of the parameters of motion found in this way will be desired evaluations.

The divergence of apriori q_a and actual q reduces Z as compared to Z_{max}. It is primarily due to the effect of the exponential factor of the subintegral expression. Indeed, if even the argument of the exponent is similar at all points in the area of integration, then for all $k\Delta r \neq 2n\pi$, this factor will be less than its maximum value, which is equal to one. In the overwhelming majority of cases, however, the difference in ranges Δr is not identical for different points of the antenna and does not remain constant during signal reception. This all brings about a reduction in ACF as compared to the phase coincidence of received signal with reference; under conditions of variability of the difference in ranges, the degree of reduction will be as great as the magnitude of the range difference. This may be clearly seen if we examine the reception of an unmodulated signal by a nondirectional antenna, where the range difference Δr fluctuates in time at a constant rate $\Delta r = \Delta r_n + \Delta vt$. By substituting this expression in formula (2.3.2) and assuming that signal amplitude is not time-dependent, we derive the following equation for ACF in the non-feedback mode of operation:

$$Z = E\frac{\sin 0,5\,k\Delta v\,T}{0,5\,k\Delta v\,T}, \qquad (2.3.5)$$

where T -- signal duration.

With an increase in the product of ΔVT, we detect in the ACF /55 a series of maximums and zeroes, while the magnitude of maximums gradually decreases. Zeroes occur when

$$\Delta v T = n\lambda ,$$

where λ is wave length, $n = 1,2,\ldots.$

ACF will have the same properties even with a more complex law of fluctuation in the difference of apriori and actual range, if only the limits of fluctuation of this quantity exceed the wave length.

4. The generalized autocorrelation function, strictly speaking, has no central symmetry. Indeed, by substituting in (2.3.4) in place of Δq_ℓ the quantity $-\Delta q_\ell$ and replacing the variables in the formula

$$t' = t - \frac{2}{v_{gr}} \sum_{l=1}^{m} \frac{\partial r}{\partial q_l} \Delta q_l ,$$

we derive

$$Z(-\Delta q) = \frac{1}{2} \left| \iint_V \int_{t_2}^{t_3} \frac{\overline{A}_a^*(t')}{1 + \frac{2}{v_{gr}} \sum_{l=1}^{m} \frac{\partial r}{\partial q_l} \Delta q_l} \right.$$

$$\left. \times \overline{A}\left(t' + \frac{2}{v_{gr}} \sum_{l=1}^{m} \frac{\partial r}{\partial q_l} \Delta q_l \right) \exp\left(i2k \sum \frac{\partial r}{\partial q_l} \Delta q_l \right) dV\, dt' \right| , \qquad (2.3.6)$$

where t_2 and t_3 -- moments of appearance and disappearance of signal at reception point.

Hence it follows that $Z(-\Delta q) \neq Z(\Delta q)$.

But because

$$\sum_{l=1}^{m} \frac{\partial r}{\partial q_l} \Delta q_l \ll v_{gr} \quad \text{and} \quad \sum_{l=1}^{m} \frac{\partial r}{\partial q_l} \Delta q_l \ll r_a ,$$

then for the autocorrelation function we may write the following expression:

$$Z(-\Delta q)=\frac{1}{2}\left|\int\limits_{V}\int\limits_{T}\overline{A}_{n}^{*}(t)\overline{A}\left(t+\frac{2}{v_{gr}}\sum_{l=1}^{m}\frac{\partial r}{\partial q_{l}}\Delta q_{l}\right)\times\right.$$
$$\left.\times\exp\left(i2k\sum_{l=1}^{m}\frac{\partial r}{\partial q_{l}}\Delta q_{l}\right)dV\,dt\right|, \qquad (2.3.7)$$

whence it follows that the autocorrelation function is an even function of the vectorial argument Δq, because (2.3.7) and (2.3.4) are moduloes of complex-conjugated quantities. /56

The comparison of autocorrelation functions for signals having variable and constant parameters indicates that, in addition to common properties, there exist several distinctions between them.

In particular, our attention is drawn to the fact that the second term of the argument of complex signal amplitude

$$\frac{2}{v_{gr}}\sum_{l=1}^{m}\frac{\partial r}{\partial q_{l}}\Delta q_{l},$$

appearing beneath the ACF integral, in the given case is not only a function of definable parameters, but is also time-dependent; the time-dependence, which allows for the term $dr(t)/dq_{\ell}$, is generally non-linear in nature.

We shall suggest that the similar argument of the ACF of a signal having constant parameters is associated with the argument of a signal by a linear relationship and is not time-dependent:

$$Z(\tau,\ F)=\frac{1}{2}\left|\int\limits_{0}^{T}\overline{A}(t)\overline{A}\,(t-\tau)\exp\left(i2\pi Ft\right)dt\right|.$$

Chapter 3

EVALUATING THE ACCURACY OF SPACE MEASURING COMPLEXES

3.1. Algorithms for making Apriori Data more Accurate and Evaluation of Accuracy of Measurements for an Isolated Signal having Unknown Initial Phase

The fullest presentation about the accuracy of measurements is given by the aposteriori probability density of evaluations of the vector of the parameters of motion, which corresponds to a given realization or sampling of the signal-and-noise mixture [18]. As we know, it is a function of the probability density of apriori data and the intercorrelation function of the reference and received signals; for an isolated signal having random initial phase, /57 it is equal to

$$w(\mathbf{q}/y) = Kw(\mathbf{q}) \exp(-E/N_0) I_0(2Z/N_0). \tag{3.1.1}$$

We can see from the formula that the relationship of the probability density as a function of the magnitude of the parameters evaluated has a complex nonlinear nature. The distributive law is generally different from the normal one.

But in the reception of strong signals, which has the most practical value, the argument of the Bessel function in formula (3.1.1) greatly exceeds one; an approximation of this function may be given by the relationship

$$I_0(x) \approx e^x / \sqrt{2\pi x},$$

where it is clear that with large values of the argument, the denominator will have a weak effect on the variation of the function and the exponent will have the dominant role. Therefore, the aposteriori probability density may be written in the form of the following product:

$$w(\mathbf{q}/y) = K_1 w(\mathbf{q}) \exp(2Z/N_0) \exp(-E/N_0). \tag{3.1.2}$$

For small deviations of the parameters of motion from their apriori values, the exponential indexes of formula (3.1.2) may be expanded in a Taylor series in the neighborhood of apriori values. In producing this expansion, we find that

$$
\begin{aligned}
Z(\mathbf{q}) - \frac{1}{2}\mathrm{E}(\mathbf{q}) = Z(\mathbf{q}_a) - \frac{1}{2}\;(\mathbf{q}_a) + \sum_i \Big[Z'_i(\mathbf{q}_a) - \\
- \frac{1}{2}\mathrm{E}'_i(\mathbf{q}_a) \Big] (q_i - q_{ai}) + \frac{1}{2}\sum_{i,j}\Big[Z''_{ij}(\mathbf{q}_a) - \\
- \frac{1}{2}\mathrm{E}''_{ij}(\mathbf{q}_a) \Big] (q_i - q_{ai})(q_j - q_{aj}) + \dots
\end{aligned}
\qquad (3.1.3)
$$

Here q_a is the apriori value of the vector of evaluated parameters; q_{ai} is the i^{th} component of this vector; $Z'_i(q_a)$ -- the value of the first derivative of the ACF with respect to the i^{th} component of the vector of evaluated parameters at point $q = q_a$; $Z''_{ij}(q_a)$ -- the value of the second derivative of the ACF with respect to the i^{th} and j^{th} components at the same point.

The maximum of the correlation integral generally does not coincide with a point corresponding to apriori data, and thus the first derivative differs from zero.

If the difference between apriori and actual values of the parameters of motion is not very great, the correlation integral may be accurately enough approximated by three terms of the Taylor series. This means that the conventional probability density of reception of a given realization is represented by a Gaussian error curve and the distribution is normal. This case deserves more thorough examination, since simple analytic relationships may be derived which describe the resulting accuracy of measurements, and algorithms for defining correction factors for apriori values of the parameters of motion. /58

Let us state that errors in apriori data conform to the normal distributive law and the probability density is depicted by the formula

$$
w(\mathbf{q}) = C\exp\left[-\frac{1}{2}(\mathbf{q}-\mathbf{q}_a)^T \mathbf{B}_a^{-1}(\mathbf{q}-\mathbf{q}_a)\right], \qquad (3.1.4)
$$

where q_a -- the mathematical expectation of the vector of apriori values of the parameters of motion; B_a -- the correlation matrix of erros in apriori data.

By substituting (3.1.3) and (3.1.4) into (3.1.2), we derive the following vectorial relationship:

$$w(\mathbf{q}/y) = K_2 \exp\Big\{-\frac{1}{2}(\mathbf{q}-\mathbf{q}_a)^{\mathrm{T}}\, \mathbf{B}_a^{-1}(\mathbf{q}-\mathbf{q}_a) - -\frac{1}{N_0}[\mathrm{E}(\mathbf{q}_a) - 2Z(\mathbf{q}_a)] + \frac{2}{N_0}(\mathbf{q}-\mathbf{q}_a)\Big[\mathbf{Z}'(\mathbf{q}_a) - \frac{1}{2}\mathrm{E}'(\mathbf{q}_a)\Big] + + \frac{1}{N_0}(\mathbf{q}-\mathbf{q}_a)^{\mathrm{T}}\Big[\mathbf{Z}''(\mathbf{q}_a) - \frac{1}{2}\mathrm{E}''(\mathbf{q}_a)\Big](\mathbf{q}-\mathbf{q}_a)\Big\}, \tag{3.1.5}$$

which in expanded form may be rewritten thus:

$$w(q_1, q_2, \ldots, q_m/y) = K_2 \exp\Big\{-\frac{1}{2}\sum_{i,j}(q_i - q_{ai})\, B_{aij}^{-1}(q_j - q_{aj}) - -\frac{1}{N_0}[\mathrm{E}(q_{a1}, q_{a2}, \ldots, q_{am}) - 2Z(q_{a1}, q_{a2}, \ldots, q_{am})] + \frac{2}{N_0}\sum_i (q_i - - q_{ai})\Big[Z_i'(q_{a1}, q_{a2}, \ldots, q_{am}) - \frac{1}{2}\mathrm{E}_i'(q_{a1}, q_{a2}, \ldots, q_{am})\Big] + + \frac{1}{N_0}\sum_{i,j}(q_i - q_{ai})\Big[Z_{ij}''(q_{a1}, q_{a2}, \ldots, q_{am}) - -\frac{1}{2}\mathrm{E}_{ij}''(q_{a1}, q_{a2}, \ldots, q_{am})\Big](q_j - q_{aj})\Big\}.$$

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In this respect, the formula for the aposteriori probability density may be reduced to the form

$$w(\mathbf{q}/y) = K_2 \exp\Big[-\frac{1}{2}(\mathbf{q}-\hat{\mathbf{q}})^{\mathrm{T}}\, \mathbf{B}^{-1}(\mathbf{q}-\mathbf{q})\Big], \tag{3.1.6}$$

where $\hat{q}$ -- the value of the vector of parameters at which the maximum aposteriori probability density is achieved.

Because the aposteriori distribution is symmetric, $\hat{q}$ is the optimum evaluation corresponding to both of the earlier noted loss functions. Consequently, the vector $\hat{q}$ may be called the vector of precise values of the parameters of motion. The letter B designates the correlation matrix of resultant measurement errors. It is a quadratic matrix, whose dimension is defined by the dimension of the vector of definable parameters.

After carrying out the appropriate transforms, we derive the following equations for the matrix B and vector-column $\hat{q}$:

$$\mathbf{B}^{-1} = \mathbf{B}_a^{-1} - \frac{2}{N_0}\left[\mathbf{Z}''(\mathbf{q}_a) - \frac{1}{2}\,\mathbf{E}''(\mathbf{q}_a)\right], \tag{3.1.7}$$

$$\hat{\mathbf{q}} = \mathbf{q}_a + \frac{2}{N_0}\,\mathbf{B}\left[\mathbf{Z}'(\mathbf{q}_a) - \frac{1}{2}\,\mathbf{E}'(\mathbf{q}_a)\right], \tag{3.1.8}$$

which may likewise be written as

$$\| B_{ij}^{-1} \| = \| B_{aij}^{-1} \| - \frac{2}{N_0}\left[\| Z_{ij}''(\mathbf{q}_a) \| - \frac{1}{2}\,\| \mathbf{E}_{ij}''(\mathbf{q}_a) \|\right],$$

$$\| \hat{q}_i \| = \| q_{ai} \| + \frac{2}{N_0}\,\| B_{ij} \|\left[\| Z_j'(\mathbf{q}_a) \| - \frac{1}{2}\,\| \mathbf{E}_j'(\mathbf{q}_a) \|\right].$$

Here $Z''(q_a) = ||Z_{ij}''(q_a)||$ -- the matrix of second derivatives of the correlation integral with respect to definable parameters of motion (derivatives are calculated at points corresponding to apriori data); $Z'(q_a) = ||Z_j'(q_a)||$ -- the vector-column of first derivatives of the correlation integral with respect to definable parameters at the same points. /60

Formulas (3.1.7) and (3.1.8) are algorithms of optimum filtration of signals having regularly variable parameters, which allow us to define, in terms of a given realization of the noise and signal mixture, the magnitudes of all components of the vector of measurable parameters of motion and to evaluate the resultant accuracy of measurement. It is clear that correction factors for apriori data and the correlation matrix of resultant error are defined by the correlation integral or, more precisely, by derivatives of the correlation integral with respect to the definable parameters of motion. Consequently, the set of the first and second derivative from the correlation integral contains very complete information both about the desired correction factors and their accuracy.

Without a doubt, a noteworthy aspect of the formulas cited is the possible distinct definition of correction factors for all definable parameters of motion. This is an interesting fact, since the question is of the vector of corrections whose dimension exceeds one, although at the output of the optimum filter, as a result of measurements, we obtain only one value of voltage equal to the definable value of the autocorrelation function. It is clear that the possible elimination of apparent indeterminacy is hidden in the excess measuring information on one hand, and in the use of apriori data on the other.

Let us now touch upon the properties of the correlation matrix of resultant measurement errors (3.1.7). If we were to call the matrix elements which are the inverse of the correlation matrix of erros "measures of accuracy", then the meaning of formula (3.1.7) may be expressed as follows. The measure of accuracy of measurement results is equal to the sum of measures of accuracy of apriori data and derived measurements. It appears that if the accuracy of apriori data is too small, the first components of the matrix will be similar to zero and accuracy will be defined by the measuring system; conversely, with low accuracy of the measuring devices, the specific gravity of the second components will be small and the resultant accuracy will correspond to the accuracy of apriori data.

In formulas (3.1.7) and (3.1.8) appear derivatives from the correlation integral at the point corresponding to apriori data. The correlation integral is a function which diminishes in proportion to an increase in the difference between apriori and actual values of the parameters of motion. Therefore, in proportion to the increase in the difference of vectors q_a and q, /61 there is a growth in the first and decrease in the second derivative of the correlation integral. This corresponds to an increase of the desired correction and a reduction of measurement accuracy. Conversely, in proportion to the approximation of the vectors mentioned, there is an increase in the second derivative and an increase in measurement accuracy. The maximum accuracy will be achieved if apriori data are taken as equal to the actual values of the measurable parameters.

Therefore, if in matrix (3.1.7), instead of values of second derivatives of ACF at the point corresponding to apriori data, we substitute the values of the second derivatives corresponding to the actual values of definable parameters, the derived matrix will allow us to judge the ultimately attainable or, as it is customarily called, the potential accuracy of measurements. ACF derivatives calculated at points corresponding to actual data which, as we know, coincide with the coordinates of the maximum correlation integral, will be designated with the symbol $Z''_{ij}(0)$, implying under the argument of the integral the difference of apriori and actual data.

It follows from formula (3.1.7) that the correlation matrix of measurements describing the potential accuracy of measurements has the form

$$B^{-1} = B_a^{-1} - Z''(0)/N_0 = B_a^{-1} - E''/N_0, \qquad (3.1.9)$$

where Z"(0) -- the maximum value of the second derivative of the signal autocorrelation function.

The aforesaid indicates the advisability of using automatic parameter measurement systems having feedback with apriori data, i.e., systems in which, as measuring information is accumulated, apriori data are continuously updated.

It is also useful to focus our attention on the fact that, in the event signals having regularly variable parameters are being received, the most complete information about evaluable parameters of motion is contained in the correlation integral or in some function of it. In view of this, the data which are most important as concerns the definition of corrections for apriori data and evaluation of their accuracy are contained in the first and second derivatives of the correlation integral.

3.2. Algorithms of Optimum Processing of Signals Having Fluctuating Parameters /62

Up until this point, we have discussed the reception of one signal having a random initial phase, whose correlation integral has the form of (2.2.14). If a signal having random phase and amplitude or a signal having fluctuating phse or fluctuating amplitude and phase is implemented, the signal processing procedure becomes complicated.

The ratio of probability for a signal having regularly variable amplitude and fluctuating phase is expressed by the formula

$$l = \prod_k \exp(-E_k/N_0) I_0(2Z_k/N_0), \qquad (3.2.1)$$

which assigns the definition of the weighted product of values of the Bessel function from correlation integrals calculated for each coherence interval (correlation interval). In practice, however, instead of calculating the ratio of probability, it is preferable to calculate the logarithm from it. The replacement of the given function by the logarithmic one is admissible in view of the monotony of the latter. In switching to the logarithm we can avoid calculating the product, by replacing it with the summation:

$$\ln l = \sum_k \ln I_0(2Z_k/N_0) - \sum_k E_k/N_0 . \qquad (3.2.2)$$

The calculation of such a summation is not difficult, since the operation of defining the logarithm of the Bessel function may be placed on a nonlinear element having the appropriate characteristics. We may also note that for strong and weak signals, the Bessel function logarithm is approximated by linear and quadratic functions, respectively

$$\ln I_0(x) \approx x,\ x \gg 1;\ \ln I_0(x) \approx x^2/4;\ x \ll 1,$$

attesting to the possible replacement of the nonlinear operation by a linear or quadratic detection. Therefore, if the specifications of the definition of the correlation integral are not borne in mind, we may consider that the signal processing procedure for a signal having regularly variable parameters formally coincides with the procedure for processing signals having constant parameters.

For a signal having fluctuating amplitude and phase (independent fluctuations), the ratio of probability is equal to /63

$$l = \prod_k \frac{N_0}{E_k + N_0} \exp\left(\frac{1}{N_0}\frac{Z_k^2}{N_0 + E_k}\right), \qquad (3.2.3)$$

and it is clear that in the given instance, we may switch to the calculation of the logarithm of the ratio of probability

$$\ln l = \frac{1}{N_0}\sum_k \frac{Z_k^2}{N_0 + E_k} + \sum_k \ln \frac{N_0}{N_0 + E_k} . \qquad (3.2.4)$$

Therefore, the procedure for defining the ratio of probability also reduces to a weighted summation of voltage at the output of a quadratic detector which follows the correlation circuit designed to define Z_k.

The algorithms (3.2.2) and (3.2.4) guarantee the optimum processing of a signal upon its detection and during the measurement process. But if we have rather reliable apriori data, the indicated algorithms should be transformed to permit us to directly judge the magnitude of corrections and measurement accuracy, i.e., it is desirable to derive from them algorithms similar to (3.1.7) and (3.1.8). We shall cite these algorithms.

It appears that if the amplitude of a signal does not fluctuate and the ratio of probability is expressed by formula (3.2.1), then with the reception of a strong signal the same laws will be in effect as those which occur in the reception of an isolated signal having a constant initial phase, whose magnitude is random. Therefore, in this case, to define corrections and evaluate the accuracy, we may use algorithms (3.1.7) and (3.1.8), by substituting in place of the derivatives of the correlation integral and energy

$$Z_i'(\mathbf{q}_a) - \frac{1}{2}\,\mathrm{E}_i'(\mathbf{q}_a), \quad Z_{ij}''(\mathbf{q}_a) - \frac{1}{2}\,\mathrm{E}_{ij}''(\mathbf{q}_a)$$

of the summation of derivatives from the correlation integrals and energy, taken within the signal coherence interval

$$\sum_k \left[Z_{ki}'(\mathbf{q}_a) - \frac{1}{2}\,\mathrm{E}_{ki}'(\mathbf{q}_a) \right],$$
$$\sum_k \left[Z_{kij}''(\mathbf{q}_a) - \frac{1}{2}\,\mathrm{E}_{kij}''(\mathbf{q}_a) \right]. \qquad (3.2.5)$$

To derive algorithms for processing a signal having fluctuating phase and amplitude, we will use a method similar to that used in section 3.1. /64

The aposteriori probability distribution density of parameters of motion for independent fluctuations in phase and amplitude of a signal may be reduced to the following form:

$$w(\mathbf{q}/y) = Kw(\mathbf{q})l(y/\mathbf{q}) = Kw(\mathbf{q})\exp[\ln l(y/\mathbf{q})] =$$
$$= K\exp\left[-\frac{1}{2}\sum_{i,j}(q_i - q_{ai})B_{aij}^{-1}(q_j - q_{aj})\right]\times$$
$$\times\exp\left(\frac{1}{N_0}\sum_k \frac{Z_k^2}{N_0 + \mathrm{E}_k} + \sum_k \ln\frac{N_0}{N_0 + \mathrm{E}_k}\right).$$

As before, let us assume that the actual values of the parameters of motion are rather similar to the apriori known values of these parameters. In this case, we may consider that $E_k(q) \simeq E_k(q_a)$ and the square of the ACF is represented by a Taylor series:

$$\begin{aligned} Z_k^2(\mathbf{q}) &= Z_k^2(\mathbf{q}_a) + 2Z_k(\mathbf{q}_a)\sum_i Z'_{ki}(\mathbf{q}_a)(q_i - q_{ai}) + \\ &+ \sum_{i,j}[Z'_{ki}(\mathbf{q}_a)Z'_{kj}(\mathbf{q}_a) + Z_k(\mathbf{q}_a)Z''_{kij}(\mathbf{q}_a)](q_i - q_{ia})(q_j - q_{aj}) + \ldots \end{aligned} \tag{3.2.7}$$

Therefore, for the aposteriori probability density of definable parameters of motion we derive the expression

$$\begin{aligned} w(\mathbf{q}/y) &= \text{const}\exp\left\{-\frac{1}{2}\left[\sum_{i,j}(q_i - q_{ai})B_{ij}^{-1}(q_j - q_{aj}) - \right.\right. \\ &\left.\left. - \frac{1}{N_0}\sum_k \frac{4}{N_0 + E_k} Z_k(\mathbf{q}_a)\sum Z'_{ki}(\mathbf{q}_a)(q_i - q_{ai})\right]\right\}, \end{aligned} \tag{3.2.8}$$

where

$$\begin{aligned} B_{ij}^{-1} = B_{aij}^{-1} - \frac{2}{N_0}\sum_k \frac{1}{N_0 + E_k}[Z'_{ki}(\mathbf{q}_a)Z'_{kj}(\mathbf{q}_a) + \\ + Z_k(\mathbf{q}_a)Z''_{kij}(\mathbf{q}_a)] \end{aligned} \tag{3.2.9}$$

or in matrix form

$$\begin{aligned} \mathbf{B}^{-1} = \mathbf{B}_a^{-1} - \frac{2}{N_0}\sum_k \frac{1}{N_0 + E_k}\{[\mathbf{Z}'_k(\mathbf{q}_a)][\mathbf{Z}'_k(\mathbf{q}_a)]^T + \\ + Z_k(\mathbf{q}_a)\mathbf{Z}''_k(\mathbf{q}_a)\}. \end{aligned} \tag{3.2.10}$$

This distributive density may, ultimately, be reduced to the form /65

$$w(\mathbf{q}/y) = \text{const}\exp\left\{-\frac{1}{2}\left[\sum_{i,j}(q_i - \hat{q}_i)B_{ij}^{-1}(q_j - \hat{q}_j)\right]\right\}, \tag{3.2.11}$$

where

$$\hat{q}_i = q_{ai} + \frac{2}{N_0} \sum_k \frac{1}{N_0 + E_k} Z_k(q_a) \sum_j B_{ij} Z'_{kj}(q_a) \tag{3.2.12}$$

or in matrix form

$$\hat{q} = q_a + \frac{2}{N_0} \sum_k \frac{1}{N_0 + E_k} Z_k(q_a) BZ'_k(q_a). \tag{3.2.13}$$

Let us note that whereas for an isolated signal the aposteriori distribution is Gaussian only for a strong signal whose energy exceeds the spectral density of noise, for a signal having fluctuating phase and amplitude, the distribution is Gaussian both for large and for small values of the signal-to-noise ratio.

Let us discuss the typical features of the derived algorithms. Similarly to what we encountered in discussing the process of processing isolated signal, the basic constituent part of these algorithms is the correlation integral and its derivatives with respect to definable parameters of motion. But, since for the isolated signal it was assumed that its phase does not fluctuate, the duration of the definition of the correlation integral was made equal to that of the measurements. More generally, when the phase of a signal fluctuates, the duration of definition of the correlation integral must be made equal to the duration of the phase correlation interval. Moreover, we likewise have to somewhat modify the procedure for defining corrections and the error correlation matrix. An examination of formulas in (3.2.13) indicates that the correction for apriori value of the vector of the orbital parameters is herewith defined by the weighted summation of corrections calculated within each correlation interval of fluctuations of phase in conformity with the formula

$$\frac{2}{N_0} BZ'_k(q_a).$$

which is a part of the similar formula (3.1.8) for an isolated signal. This relationship is identical to the formula for the correction which is derived using the method of least squares. /66
The weight coefficients used in summation of corrections for discrete correlation intervals are the ratios

$$Z_k(q_a)/(N_0 + E_k) = Z_k(\Delta q)/(N_0 + E_k), \tag{3.2.14}$$

whose numerical values are included between zero and one.

The values of the autocorrelation functions are a function of the differences between apriori and actual values of the parameters of motion. Since, in conformity to the customary assumption of the differences of $q-q_a$ are small, the spectral density of noise and the signal energy are virtually independent of these differences. Therefore, the magnitude of the weight coefficient in the given correlation interval is as small as is large the magnitude of the aforementioned difference. Such nature of the relationship of weight coefficients as functions of the differences between apriori and actual values of the parameters of motion explains the absence of a factor before the summation sign which is inversely proportional to the number of correlation intervals confined within the duration of the measurement session. This type of factor, at first glance, seems necessary, since without it the resultant correction would be equal to the sum of errors in discrete correlation intervals. The role of this factor is essentially played by the aforementioned weight coefficients.

Thus, the values of the weight coefficients are functions of the difference between actual and apriori values of the parameters of motion. Moreover, as we can see from formula (3.2.14), they are functions of the signal-to-noise ratio in the correlation interval (i.e., of the ratio of signal energy in the correlation interval to the spectral density of noise), while this relationship is as strong as this ratio is small. Indeed, when $E_k >> N_0$, the weight coefficient is equal merely to the normalized value of the ACF $Z_k(q_a)/E_k = Z_{k1}(q_a)$. If, however, the signal-to-noise ratio is small, the weight coefficient is equal to the product of this same normalized ACF value multiplied by the signal-to-noise ratio

$$\frac{1}{N_0} Z_k(q_a) = \frac{1}{N_0} E_k Z_{kn}(q_a).$$

Let us now examine the distinctive features of the correlation matrix. As with an isolated signal having initial phase which is invariable during the observation session and regularly /67 variable amplitude, in this case, the matrix opposite to the error correlation matrix of measurements is equal to the sum of matrices, one of which is opposite the apriori data error correlation matrix, and one which describes the accuracy properties of the measuring complex itself. But in contrast to the previously discussed case, the accuracy properties of the complex are described by the sum of magnitudes calculated within discrete correlation intervals of phase fluctuation. In turn, the aforementioned components, calculated within the correlation intervals, consists of two components. The first of these

$$\frac{Z_k(\mathbf{q}_a)}{\mathrm{E}_k + N_0}\frac{2}{N_0}\mathbf{Z}_k''(\mathbf{q}_a)$$

is close in structure and magnitude to the component which we encounted in examining a signal having constant initial phase and regularly variable amplitude, and differs from it only in its weight coefficient, which is precisely equal to the weight coefficient which appears in the correction formula (3.2.13). The second component was previously missing. It is proportional to the product of the first derivatives of the ACF

$$\frac{[\mathbf{Z}_k'(\mathbf{q}_a)]}{N_0 + \mathrm{E}_k}\frac{[\mathbf{Z}_k'(\mathbf{q}_a)]^T}{N_0}.$$

Our interest is drawn to the correlation matrix of errors describing the potential accuracy of the complex. The highest accuracy is attained, as we know, at $q_a = q$. Consequently, after completing the process of updating the parameters of motion, the measurement accuracy will be reflected by a correlation matrix satisfying the equation

$$\mathbf{B}^{-1} = \mathbf{B}_a^{-1} - \sum_k \frac{\mathrm{E}_k}{N_0 + \mathrm{E}_k}\frac{2}{N_0}\mathbf{Z}_{k_{ij}}''(0). \qquad (3.2.15)$$

The second component of the formula describes the accuracy increment attained as a result of using the means of the measuring complex. It appears that the complex should only be put into operation if this increment is sufficiently great in comparison with the first term of formula (3.2.15). The primary constituent part of the second term, as for a signal having invariable initial phase and regularly variable amplitude, is the maximum value of the second derivative of the ACF.

A measure of the potential accuracy of the measuring system proper, described by the second term of formula (3.2.15), is defined by means of a weighted summation of measures of potential accuracy of measurements within discrete intervals of phase fluctuation correlation. The magnitude of the weight coefficients approaches one with low levels of noise and is close to the magnitudes of the signal-to-noise ratios in correlation intervals -- with high interference levels. /68

Hence it follows that with large signal-to-noise ratios in the phase fluctuation correlation interval, the increment of accuracy of data on the parameters of motion, owing to the operation of the complex means, is proportional to the first power of these ratios; at small values of the ratio of signal energy to the spectral density of noise, it is proportional to the square of those ratios. Indeed, at large ratios of signal-to-noise, the increment of accuracy described by the second term of the correlation matrix is proportional to the ratio of the second derivative of the ACF to the spectral density of noise. The second derivative may be written as the product of signal energy times the normalized value of the second derivative of the ACF:

$$\boxed{\sum_k Z''_{ijk}(0) = \sum_k \frac{E_k Z''_{ijk}(0)}{E_k} = \sum_k E_k Z''_{ijk_e}(0).}$$

Consequently,

$$\boxed{\sum_k \frac{2}{N_0} Z''_{ijk}(0) = 2\sum_k \frac{E_k}{N_0} Z''_{ijk_e}(0).}$$

By analogy with this, for small ratios of signal-to-noise we get

$$\boxed{\Delta B^{-1}_{ijk} = 2\sum_k \left(\frac{E_k}{N_0}\right)^2 Z''_{ijk_e}(0).}$$

Hence we can see the amount of gain in accuracy produced by increasing the duration of the signal phase fluctuation correlation interval. It is likewise useful to note that if signal energy in the phase fluctuation correlation interval greatly exceeds the spectral density of interference, then to evaluate the potential accuracy we may use the maximum value of the second derivative of the ACF, calculated in the time interval equal to the duration of the entire measurement session, $T = \sum_k T_k$, since in this case /69

$$\boxed{\sum_k Z''_{ijk}(0) = Z''_{ij}(0)\Big|_{T=\sum_k T_k}.}$$

The formula for the correlation matrix of resultant measurement error (3.2.15) allows us to judge the nature of the relationship between the accuracy and length of the measurement process. With rather strong signals, the measure of system accuracy increases in proportion to time. This signifies that when one parameter is measured, in proportion to the increase in time, the quantity inversely proportional to error dispersion is equal to the sum of quantities inversely proportional to the dispersions of measurements in the initial and additional segments of time. With weak signals, the increase in accuracy is retarded: the increment in accuracy is proportional not only to the second derivative of the ACF, but also to the signal-to-noise ratio in the added time interval.

A specific representation on the structure of an optimum signal filtration system in a space complex for trajectory measurements is illustrated in Fig. 3.1. This figure cites a consolidated /70
functional schematic of a system implementing the algorithms of optimum processing of isolated signals having random initial phase (3.1.7) and (3.1.8).

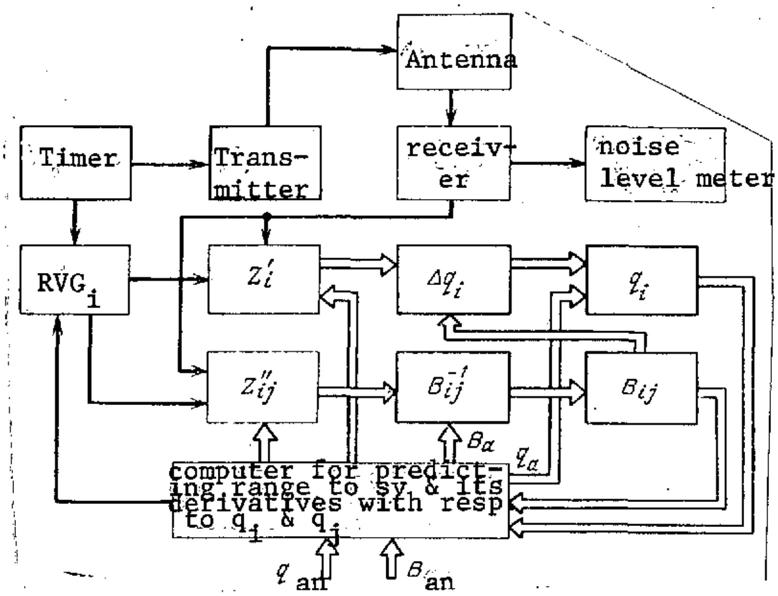

Fig. 3.1. Functional Schematic of Optimum Signal Filtration System in Space Measuring Complex.

The system contains signal processing devices in addition to the devices required for shaping and recording the fields: timer, transmitter, antenna, and receiver. The basic elements of these devices are correlation circuits, which ensure the computation of the first and second derivatives of the correlation integrals, reference voltage generators (RVG$_i$) and a computer for predicting the values of range and derivatives from predicted values of range with respect to definable parameters of motion. This machine controls the work of the reference signal generator and shapes the information needed for the operation of the correlation circuits. The circuits which calculate corrections for definable parameters of motion and correlation matrices are also integral parts of this machine. Coupled circuits in Fig. 3.1 are shown by circuits through which circulate data on vectorial quantities. In the same figure is shown a processing system for signals tapped from the one-element antenna. If the measuring complex contains several systems arranged in space, each of them must be provided with its own processing system like the system illustrated in Fig. 3.1. Some elements may be common for the entire complex. They are indicated in the figure by circuits having thick lines. It goes without saying, the functional schematic of Fig. 3.1 is illustrative in nature and does not reflect the particularities of technical implementation of the processing system.

Let us note in conclusion that in computing the derivatives of signal energy with respect to the definable parameters of motion, which appear in formulas 3.1 and 3.2, there are usually no problems encountered. These derivatives are expressed by the formulas

$$E'_i = \frac{\partial E}{\partial q_i} = \frac{1}{2} \int_V \int_T \frac{\partial r}{\partial q_i} A A'_r \, dV \, dt,$$

$$E''_{ij} = \frac{1}{2} \int_V \int_T \left\{ \frac{\partial^2 r}{\partial q_i \partial q_j} A A'_r + \frac{\partial r}{\partial q_i} \frac{\partial r}{\partial q_j} [A A''_r + (A'_r)^2] \right\} dV \, d$$

As pertains to partial derivatives from the autocorrelation function, we should examine the methods for computing them in greater detail.

3.3. Derivatives of Autocorrelation Functions of Signals Having Regularly Variable Parameters /71

Differentiation of the autocorrelation function of a signal having rnadom initial phase is associated with some problems. The cause of these problems is the fact that the derivative of the modulus of a complex function is not equal to the modulus of its derivative. Therefore, prior to differentiating we must calculate the modulus, and only the thus derived weighted function may be subjected to differentiation. To simplify computations, let us introduce the following notations:

$$Z = \frac{1}{2} \left| \int_V \int_T \overline{A}^*_a \left(t + \frac{2\Delta r}{v_{gr}} \right) \overline{A}(t) \times \exp(-i \cdot 2k \Delta r) \, dV \, dt \right| = \frac{1}{2} |I_1 + i I_2| . \qquad (3.3.1)$$

where

$$I_1 = \int\limits_V \int\limits_T A\left(t + \frac{2\Delta r}{v_{gr}}\right) A(t) \cos \psi(\Delta r)\, dV\, dt; \quad (3.3.2)$$

$$I_2 = \int\limits_V \int\limits_T A\left(t + \frac{2\Delta r}{v_{gr}}\right) A(t) \sin \psi(\Delta r)\, dV\, dt; \quad (3.3.3)$$

$$\psi(\Delta r) = -\varphi\left(t + \frac{2\Delta r}{v_{gr}}\right) + \varphi(t) - 2k\Delta r; \quad \Delta r = \Delta r(\mathbf{q}_a, \mathbf{q}). \quad (3.3.4)$$

Taking these notations in account, the formula for the auto-correlation function is

$$Z = \frac{1}{2}\sqrt{I_1^2 + I_2^2}\,. \quad (3.3.5)$$

Let us differentiate the derived weighted function with respect to q_{ai} and q_{aj}:

$$Z_i' = \frac{1}{2}\,\frac{I_1 I_{1i}' + I_2 I_{2i}'}{\sqrt{I_1^2 + I_2^2}}, \quad (3.3.6)$$

$$Z_{ij}'' = \frac{1}{2}\,\frac{1}{\sqrt{I_1^2 + I_2^2}}\left(I_{1i}' I_{1j}' + I_1 I_{1ij}'' + I_{2j}' I_{2i}' + I_2 I_{2ij}''\right) - \frac{1}{2}\,\frac{1}{\sqrt{(I_1^2 + I_2^2)^3}}\left(I_1 I_{1i}' + I_2 I_{2i}'\right)\left(I_1 I_{1j}' + I_2 I_{2j}'\right). \quad (3.3.7)$$

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Let us compute the first and second derivatives of the corresponding integrals and values of these derivatives at a point where the apriori and actual values of definable parameters of motion are equal to each other, i.e., they correspond to the zero value of corrections for unknown parameters or zero value of range differences.

Let us mention that the variables herewith are the apriori values of the parameters of motion, which under other equal conditions define the clearly defined (in formulas 3.3.3 and 3.3.4) magnitude of difference in apriori and actual range: $r = r - r_a$.

In the formulas derived in differentiation, there appear partial derivatives of apriori ranges with respect to the apriori values of the parameters of motion $\partial r_a/\partial q_{ai}$. Henceforth these derivatives will be denoted by the symbol $\partial r/\partial q_i$.

Partial derivatives of the following form will also be encountered in the formulas

$$\frac{\partial A_a(t+2\Delta r/v_{gr})}{\partial r_a}.$$

It is easy to ascertain that they are associated with the derivatives of these same quantities with respect to time by the equations

$$\frac{\partial A_a(t+2\Delta r/v_{gr})}{\partial r_a}=\frac{\partial A_a(t+2\Delta r/v_{gr})}{\partial(t+2\Delta r/v_{gr})}\,\frac{\partial(t+2\Delta r/v_{gr})}{\partial r_a}\,\frac{\partial r_a}{\partial q_{ai}}.$$

Let us introduce the notations:

$$A'_t=\frac{\partial A_a(t+2\Delta r/v_{gr})}{\partial(t+2\Delta r/v_{gr})};\quad \frac{\partial(t+2\Delta r/v_{gr})}{\partial r_a}=\frac{2}{v_{gr}}.$$

In view of this we may write:

$$I'_{1i}=\frac{2}{v_{gr}}\int_V\int_T\frac{\partial r}{\partial q_i}A'_t\left(t+\frac{2\Delta r}{v_{gr}}\right)A(t)\cos\psi(\Delta r)\,dV\,dt-$$
$$-\int_V\int_T A\left(t+\frac{2\Delta r}{v_{gr}}\right)A(t)\sin\psi(\Delta r)\psi'_i(\Delta r)\,dV\,dt, \qquad (3.3.8)$$

$$I'_{2i}=\frac{2}{v_{\mathrm{gr}}}\int\limits_V\int\limits_T \frac{\partial r}{\partial q_i} A'_i\left(t+\frac{2\,\Delta r}{v_{\mathrm{gr}}}\right) A(t)\ \sin\ \psi(\Delta r)\, dV\, dt +$$

$$+\int\limits_V\int\limits_T A\left(t+\frac{2\,\Delta r}{v_{\mathrm{gr}}}\right) A(t)\,\psi'_i(\Delta r)\cos\psi(\Delta r)\, d\,V\,dt, \qquad (3.3.9)$$ /73

$$I''_{1ij}=\frac{4}{v^2_{\mathrm{gr}}}\int\limits_V\int\limits_T \frac{\partial r}{\partial q_i}\ \frac{\partial r}{\partial q_j}\ A''_i\left(t+\frac{2\,\Delta r}{v_{\mathrm{gr}}}\right)\times$$

$$\times A(t)\ \cos\ \psi(\Delta r)\, d\,V\,dt-$$

$$-\frac{2}{v_{\mathrm{gr}}}\int\limits_V\int\limits_T\left[\frac{\partial r}{\partial q_i}\ \psi'_j(\Delta r)+\frac{\partial r}{\partial q_j}\ \psi'_i(\Delta r)\right]\times$$

$$\times A'_i\left(t+\frac{2\,\Delta r}{v_{\mathrm{gr}}}\right)\sin\ \psi(\Delta r)\, A(t)\, dV\, dt-$$

$$-\int\limits_V\int\limits_T A\left(t+\frac{2\,\Delta r}{v_{\mathrm{gr}}}\right) A(t)\,\psi'_i(\Delta r)\,\psi'_j(\Delta r)\ \cos\ \psi(\Delta r)\, dV\, dt-$$

$$-\int\limits_V\int\limits_T A\left(t+\frac{2\,\Delta r}{v_{\mathrm{gr}}}\right) A(t)\,\psi''_{ij}(\Delta r)\ \sin\ \psi(\Delta r)\, d\,V\,dt+$$

$$+\frac{2}{v_{\mathrm{gr}}}\int\limits_V\int\limits_T \frac{\partial^2 r}{\partial q_i\,\partial q_j}\, A'_i\left(t+\frac{2\,\Delta r}{v_{\mathrm{gr}}}\right) A(t)\ \cos\ \psi(\Delta r)\, d\,V\,dt, \qquad (3.3.10)$$

$$I''_{2ij}=\frac{4}{v^2_{\mathrm{gr}}}\int\limits_V\int\limits_T \frac{\partial r}{\partial q_i}\ \frac{\partial r}{\partial q_j}\ A''_i\left(t+\frac{2\,\Delta r}{v_{\mathrm{gr}}}\right)\times$$

$$\times A(t)\ \sin\ \psi(\Delta r)\, d\,V\,dt+$$

$$+\frac{2}{v_{\mathrm{gr}}}\int\limits_V\int\limits_T\left[\frac{\partial r}{\partial q_i}\ \psi'_j(\Delta r)+\frac{\partial r}{\partial q_j}\ \psi'_i(\Delta r)\right]\times$$

$$\times A'_i\left(t+\frac{2\,\Delta r}{v_{\mathrm{gr}}}\right) A(t)\ \cos\ \psi(\Delta r)\, d\,V\,dt+$$

$$+\int\limits_V\int\limits_T A\left(t+\frac{2\,\Delta r}{v_{\mathrm{gr}}}\right) A(t)\,\psi''_{ij}(\Delta r)\ \cos\ \psi(\Delta r)\, d\,V\,dt-$$

$$-\int_V\int_T A\left(t+\frac{2\Delta r}{v_{gr}}\right)A(t)\psi_i'(\Delta r)\psi_j'(\Delta r)\sin\psi(\Delta r)\,dV\,dt+$$

$$+\frac{2}{v_{gr}}\int_V\int_T\frac{\partial^2 r}{\partial q_i\partial q_j}A_t'\left(t+\frac{2\Delta r}{v_{gr}}\right)A(t)\sin\psi(\Delta r)\,dV\,dt,$$

(3.3.11) /74

$$\psi_i'(\Delta r)=-\varphi_t'\left(t+\frac{2\Delta r}{v_{gr}}\right)\frac{2}{v_{gr}}\frac{\partial r}{\partial q_i}-2k\frac{\partial r}{\partial q_i},$$

$$\psi_{ij}''(\Delta r)=-\frac{4}{v_{gr}}\frac{\partial r}{\partial q_i}\frac{\partial r}{\partial q_j}\varphi_t''\left(t+\frac{2\Delta r}{v_{gr}}\right)-$$

$$-\frac{2}{v_{gr}}\frac{\partial^2 r}{\partial q_i\partial q_j}\varphi_t'\left(t+\frac{2\Delta r}{v_{gr}}\right)-2k\frac{\partial^2 r}{\partial q_i\partial q_j}. \quad (3.3.12)$$

By substituting these equations in the expression for the second derivative of the ACF of the signal field, we may derive a general formula for the second derivative of the ACF which describes the accuracy of measurements at any relationships between the apriori and actual values of the parameters of motion. This formula, however, is extremely unwieldy, and thus we shall only cite the formula for the maximum value of the derivative. To calculate the maximum value of the second derivative of the ACF, let us first define the values of the integrals I_1 and I_2 and their derivatives, and likewise the derivatives of phase ψ where q_a = $= q$, i.e., at points where $\Delta q = 0$:

$$I_1(0)=\int_V\int_T A^2(t)\,dV\,dt=2E \quad (3.3.13)$$

$$I_2(0)=0,\ \psi(0)=0; \quad (3.3.14)$$

$$\psi_i'(0)=-\frac{2}{v_{gr}}\frac{\partial r}{\partial q_i}\varphi_t'(t)-2k\frac{\partial r}{\partial q_i}, \quad (3.3.15)$$

$$\psi_{ij}''(0)=-\frac{4}{v_{gr}^2}\frac{\partial r}{\partial q_i}\frac{\partial r}{\partial q_j}\varphi_t''(t)-$$

$$-\frac{2\partial^2 r}{\partial q_i\partial q_j}\left[\frac{1}{v_{gr}}\varphi_t'(t)+k\right], \quad (3.3.16)$$

$$I_{1i}'(0)=\frac{2}{v_{gr}}\int_V\int_T\frac{\partial r}{\partial q_i}A'A\,dV\,dt, \quad (3.3.17)$$

$$I'_{2i}(0) = \int_V \int_T \psi'_i(0)\, A^2\, dV\, \partial t; \qquad (3.3.18)$$ /75

$$I''_{1ij}(0) = \frac{2}{v_{gr}} \int_V \int_T \frac{\partial^2 r}{\partial q_i \partial q_j} AA'\, dV\, dt + \frac{4}{v^2_{gr}} \int_V \int_T \frac{\partial r}{\partial q_i} \frac{\partial r}{\partial q_j} AA''\, dV\, dt - \int_V \int_T \psi'_i(0)\, \psi'_j(0)\, A^2\, dV\, dt, \qquad (3.3.19)$$

$$I''_{2ij}(0) = \frac{2}{v_{gr}} \int_V \int_T \left[\frac{\partial r}{\partial q_i} \psi'_j(0) + \frac{\partial r}{\partial q_j} \psi'_i(0)\right] AA'\, dV\, dt + \int_V \int_T \psi''_{ij}(0)\, A^2\, dV\, dt. \qquad (3.3.20)$$

We may now write relationships for the maximum values of the second derivatives of the autocorrelation function. By substituting the appropriate values of the integrals and their derivatives into (3.3.7), we derive

$$2Z''_{ij}(0) = \left[I''_{1ij}(0) + \frac{I'_{2i}(0)\, I'_{2i}(0)}{I_1(0)} \right] = \frac{2}{v_{gr}} \int_V \int_T \frac{\partial^2 r}{\partial q_i \partial q_j} AA'\, dV\, dt + \frac{4}{v^2_{gr}} \int_V \int_T \frac{\partial r}{\partial q_i} \frac{\partial r}{\partial q_j} AA''\, dV\, dt - \int_V \int_T \psi'_i(0)\, \psi'_j(0)\, A^2\, dV\, dt + \frac{1}{2E} \left[\int_V \int_T \psi'_i(0)\, A^2\, dV\, dt\right] \left[\int_V \int_T \psi'_j(0)\, A^2\, dV\, dt\right]. \qquad (3.3.21)$$

$$Z''_{ij}(0) = \frac{1}{v_{gr}} \int_V \int_T \frac{\partial^2 r}{\partial q_i \partial q_j} AA'_t \, dV\, dt +$$

$$+ \frac{2}{v_{gr}^2} \int_V \int_T \frac{\partial r}{\partial q_i} \frac{\partial r}{\partial q_j} AA''_t \, dV\, dt -$$

$$- \frac{2}{v_{gr}^2} \int_V \int_T \frac{\partial r}{\partial q_i} \frac{\partial r}{\partial q_j} A^2 [\varphi'_t(t)]^2 \, dV\, dt -$$

$$- 4 \frac{k}{v_{gr}} \int_V \int_T \frac{\partial r}{\partial q_i} \frac{\partial r}{\partial q_j} \varphi'_t(t) A^2 \, dV\, dt -$$

$$- 2k^2 \int_V \int_T \frac{\partial r}{\partial q_i} \frac{\partial r}{\partial q_j} A^2 \, dV\, dt +$$

$$+ \frac{1}{E} \left\{ \int_V \int_T \frac{\partial r}{\partial q_i} \left[k + \frac{1}{v_{gr}} \varphi'_t(t) \right] A^2 \, dV\, dt \right\} \times$$

$$\times \left\{ \int_V \int_T \frac{\partial r}{\partial q_j} \left[k + \frac{\varphi'_t}{v_{gr}} \right] A^2 \, dV\, dt \right\}. \qquad (3.3.22)$$

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Finally, there is a certain amount of interest in such form of presentation of secondary derivatives:

$$Z''_{ij}(0) = \frac{1}{v_{gr}} \int_V \int_T \frac{\partial^2 r}{\partial q_i \partial q_j} AA'_t \, dV\, dt +$$

$$+ \frac{2}{v_{gr}^2} \int_V \int_T \frac{\partial r}{\partial q_i} \frac{\partial r}{\partial q_j} AA'' \, dV\, dt -$$

$$-2 \int_V \int_T \frac{\partial r}{\partial q_i} \frac{\partial r}{\partial q_j} k_e^2 A^2 \, dV\, dt +$$

$$+ \frac{1}{E} \left(\int_V \int_T \frac{\partial r}{\partial q_i} k_e A^2 \, dV\, dt \right) \times$$

$$\times \left(\int_V \int_T \frac{\partial r}{\partial q_j} k_e A^2 \, dV\, dt \right), \qquad (3.3.23)$$

where $k_E = k + \phi_t'/\upsilon_{gr}$.

If the initial phase of signal field oscillation carriers is known, the integral $I_2(0)$ and its derivatives would be equal to zero and the expression for the maximum value of the second derivative of the ACF would acquire the form /77

$$Z_{ij}''(0) = \frac{1}{v_{gr}} \int\limits_V \int\limits_T \frac{\partial^2 r}{\partial q_i \partial q_j} AA' \, dV \, dt + 2 \int\limits_V \int\limits_T \frac{\partial r}{\partial q_i} \frac{\partial r}{\partial q_j} \left[\frac{AA''}{v_{gr}^2} - k^2 A^2 \right] dV \, dt. \qquad (3.3.24)$$

But, as we know, in space measuring complexes we use ultrashort waves and therefore, stabilization and definition of the initial phase of oscillation carriers is coupled with enormous technical problems. Moreover, the use of information contained in the phase of oscillation carriers, due to the heterogeneity of the results of phase measurements, is only possible in practice with the use of differential-range or angular measurements. Therefore, in examining the possibilities of space measuring complexes, we generally must base our calculations on formulas (3.3.21) and (3.3.22).

The signal envelope A(t) in practice usually possesses symmetry with respect to some moment in time. Thus, its first derivative is an odd function with respect to this moment in time and the first integral of formula (3.3.23) is equal to zero. By allowing for this, we will later, for most cases, be using formulas in which the term from the first derivative of signal amplitude will not be shown under the integral sign.

Prior to switching over to a detailed analysis of formulas for the maximum values of the secondary derivatives of the ACF, let us consider the structure of these formulas and some general properties of them.

It can be ascertained that the first two terms of formula (3.3.23) reflect information contained in the signal envelope; the third term -- allows for information supplied by the difference in phases of oscillation carriers, and likewise the information due to carrier phase modulation. In contrast to the first three terms, which describe the useful effect of field utilization, the fourth term allows for information losses occurring as a result of missing data on initial signal phase.

Analysis of formula (3.3.22) sheds some additional light on factors of which the secondary derivatives of the ACF are functions. Here may be emphasized those terms initiated by phase modulation of the emitted signal, and terms occurring as a result of phase fluctuation governed by SV motion. The third and fourth terms of formula (3.3.22) are related to the first group; the components of the subsequent term, which are functions of phase derivatives, are also related to this group. The fifth term and the components of the sixth term, which contains the wave number, are related to the second group. /78

All the terms of the formula describing the second derivative may be divided into groups by the following feature as well. In front of the terms of the first group stands the number $1/\upsilon_{gr}^2$; the terms of the second group are proportional to k/υ_{gr}; and the terms of the third group contain a coefficient equal to the square of the wave number k. It appears that the coefficients standing before terms of the first group are distinguished by the least; and before the terms of the third group -- by the greatest numerical values. If the integrals which make up the terms of the corresponding groups are similar to each other, then terms of the latter group, which contain the square of the wave number, will carry the most weight.

From the preceeding statement, it is clear that formulas (3.3.22) and (3.3.23), in conjunction with the appropriate formulas from sections 3.1 and 3.2, describe the potential accuracy of measurements. In analyzing these formulas, we must investigate the semantic content of this concept in greater detail.

We only gave a general definition of this concept before. We agreed that potential measurement accuracy would imply the highest accuracy obtained in measurements using a given signal against the background of interference, which is likewise considered given by the measuring system, which introduces no errors into the measurement results. In examining the composition of formulas (3.3.22) and (3.3.23), we may conclude that this notion characterizes the accuracy which may be achieved with complete utilization of signal resources. Formulas (3.3.22) and (3.3.23) give the most general and complete picture about information provided by all signal parameters with their efficient processing, regardless of the weight relationships of the data contained in discrete signal parameters under certain concrete conditions.

The potential accuracy which is described by general information resources of the signal electromagnetic field in a given area of space during a given interval of time we shall call the potential accuracy of measurements.

It is well known, however, that different field parameters may have different information-metric capability according to the selection of the wave range, form, law of modulation, and geometric quantities on which the given parameters provide information. On the other hand, as experience has shown, systems which differ in the parameters used differ considerably in their structural and technical characteristics. Systems are usually designed and used which have been calculated to obtain data in terms of one signal parameter. Thus, in addition to the idea of potential accuracy of measurement, which describes the information resources of the field as a whole, we should also use similar concepts for the discrete parameters of the field, as well as for different field parameters with respect to the measurement of geometric and kinematic quantities used to reflect SV motion. Therefore, henceforth, in addition to the term "potential accuracy of measurements" we will use such terms as "potential accuracy of phased range-finding methods of measurement for Cartesian topocentric coordinates", "potential accuracy of phased goniometer methods for defining Keplerian orbital elements" and the like. /79

As we know, these type of terms are being used in practice. It appears that the use of such terms does not eliminate the possible use of a more general term -- potential accuracy of measurements, since the latter not only allows the evaluation of field resources as a whole, but moreover opens ways for exposing information relationships and connections between discrete signal parameters.

Formulas (3.3.22) and (3.3.23) describe the limiting resources of electronic methods of measuring the parameters of motion, i.e., those accuracy boundaries beyond which it is impossible to pass without increasing signal energy, reducing the level of interference, or increasing the dimensions of antenna sysstems. No improvement of signal processing methods, within the framework of presentations used, will allow us to achieved a reduction in error as compared with those values which are defined by the formulas in question. Consequently, the following feature of formulas for maximum values of the secondary derivatives of the ACF is of interest.

Of those geometric quantities reflecting the conditions and methods of measurement, only instantaneous range to the SV is presented here, or more precisely, its derivatives with respect to the definable parameters of motion. Neither the velocity characteristics of motion nor the goniometer coordinates of the objects are clearly presented in the formulas, although among the definable parameters of motion we usually include both velo- /80 city and goniometer quantities. The reason for the absence of data on the velocity and angles in these formulas is associated with the fact that the primary source of data on the parameters of motion is the signal time lag proportional to the distance between the SV and the point of observation.

This does not imply, generally speaking, that information on the velocity and angular coordinates is not taken into account by the formulas in question. In reality, it is reflected in them tacitly. The consideration of speed data is done by the derivatives of range themselves, which function as time-functions and are integrated with respect to time. Angular information on SV location is included in values of the subintegral expression which are subjected to spatial integration, which is a function of the point coordinates of the antenna field and consequently, the angular coordinates of the object.

Therefore, the general formula relationships which describe the potential accuracy of measurements tacitly take into account no only information about ranges to the SV, but also information about the angular coordinates and velocities of SV motion.

The problem of reflecting angular and velocity information will be investigated in Chapter 6 in greater detail.

3.4. Vectorial Form of Writing the Maximum Values of the Second Derivatives of the Autocorrelation Function

The maximum values of the second derivatives of the ACF may be written in a more compact form, if we use vector symbols.

Let us introduce the vector-line of partial derivatives of range with respect to definable parameters of motion:

$$\frac{\partial r}{\partial \mathbf{q}} = \left\| \frac{\partial r}{\partial q_1} \quad \frac{\partial r}{\partial q_2} \quad \frac{\partial r}{\partial q_3} \quad \frac{\partial r}{\partial q_4} \quad \frac{\partial r}{\partial q_5} \quad \frac{\partial r}{\partial q_6} \right\|. \qquad (3.4.1)$$

The product matrix of partial derivatives of range with respect to definable parameters of motion in the given notations may be written as follows: /81

$$\left\| \frac{\partial r}{\partial q_i} \quad \frac{\partial r}{\partial q_j} \right\| = \left[\frac{\partial r}{\partial \mathbf{q}}\right]^{\mathrm{T}} \frac{\partial r}{\partial \mathbf{q}}. \qquad (3.4.2)$$

Consequently, the matrix of maximum values of the second derivatives of the ACF acquires the form

$$Z_q''(0) = \frac{2}{v_{gr}^2} \int_V \int_T \left[\frac{\partial r}{\partial q}\right]^T \frac{\partial r}{\partial q} AA'' \, dV \, dt -$$

$$- 2 \int_V \int_T \left[\frac{\partial r}{\partial q}\right]^T \frac{\partial r}{\partial q} k_e^2 A^2 \, dV \, dt +$$

$$+ \frac{1}{E} \left(\int_V \int_T \left[\frac{\partial r}{\partial q}\right]^T k_e A^2 \, dV \, dt \right) \left(\int_V \int_T \frac{\partial r}{\partial q} k_e A^2 \, dV \, dt \right).$$

(3.4.3)

Let us further use the vectorial form of writing the instantaneous range from the point of observation to the space vehicle.

We will introduce the topocentric system of coordinates which in orientation may be inertial, Greenwich, meridional or any other. In this system, SV location will be represented as a radius-vector

$$x = \| x_1 \, x_2 \, x_3 \|^T,$$

and the distance from the observation point to the SV -- by the length of this radius-vector, which is equal to

$$r = \sqrt{x_1^2 + x_2^2 + x_3^2} = | x^T x |^{1/2}. \tag{3.4.4}$$

The vector-column of partial derivatives of range with respect to the parameters of motion has the following form:

$$\frac{\partial r}{\partial q} = \frac{\partial}{\partial q} [x^T x]^{1/2} = \frac{1}{r} x^T \frac{\partial x}{\partial q}. \tag{3.4.5}$$

Consequently, the product matrix of partial derivatives of range is represented by the formula

$$\left\| \frac{\partial r}{\partial q_i} \; \frac{\partial r}{\partial q_j} \right\| = \left[\frac{\partial r}{\partial q}\right]^T \frac{\partial r}{\partial q} = \frac{1}{r^2} \left[\frac{\partial x}{\partial q}\right]^T x x^T \frac{\partial x}{\partial q}. \tag{3.4.6}$$

Therefore, the matrix of maximum values of the secondary derivatives of the ACF with respect to definable parameters of motion is equal to

$$Z''_{q}(0) = 2\int\limits_{V}\int\limits_{T}\frac{1}{r^2}\left[\frac{\partial x}{\partial q}\right]^{T} xx^{T}\frac{\partial x}{\partial q}\left(\frac{1}{v^2_{gr}}AA'' - k^2_e A^2\right)dV\,dt + \\ + \frac{1}{E}\left(\int\limits_{V}\int\limits_{T}\frac{1}{r}\left[\frac{\partial x}{\partial q}\right]^{T} x\,k_e A^2\,dV\,dt\right)\left(\int\limits_{V}\int\limits_{T}\frac{1}{r}x^{T}\frac{\partial x}{\partial q}k_e \times \\ \times A^2\,dV\,dt\right). \qquad (3.4.7)$$

Let us note that this form of writing the second derivative of the ACF is valid only if SV motion is given in a topocentric system of coordinates.

Formula (3.4.7) must be appropriately transformed if the origin of the system of coordinates is transposed to a point in space which does not coincide with the point of observation.

Chapter 4

ANALYSIS OF POTENTIAL ACCURACY OF DIFFERENT METHODS OF MEASURING THE PARAMETERS OF MOTION

4.1. In Place of an Introduction

In this chapter we are planning to give an analysis of the formulas cited in section 3.3 for the maximum values of the second derivatives of the ACF with respect to definable parameters of motion. This analysis should begin with a discussion of a most simple application of these formulas -- the evaluation of potential accuracy of measuring the parameters of motion of a uniformly moving object. To simplify the formulas and avoid spatial integration which in this case is of no theoretical value, we will assume that the signal is received by a one-element nondirectional antenna. Let us assume, moreover, that the function which defines the law of signal phase modulation is selected so its first derivative is an odd function of time. Finally, let us consider that /83
the amplitude of the received signal is an even function of time, and the inception of time reference coincides with the location of the axis of symmetry of the received signal envelope.

We will show that in the particular instance of evaluation of initial phase to an object and the constant rate of motion, relationships (3.3.22) and (3.3.23) are reduced to derivatives of formulas of Woodword's indeterminate form known from the literature. Indeed, by assuming that

$$r = r_n + vt \tag{4.1.1}$$

and taking into account that

$$\partial r/\partial r_n = 1;\ \psi'_{r_n} = -2\phi'/\upsilon_{gr} - 2k = -2k_e;$$

$$\partial r/\partial v = t;\ \partial^2 r/\partial r_n \partial v = 0;\ \psi'_v = -2k_e t, \tag{4.1.2}$$

we find for the maximum value of the second derivative of the ACF with respect to initial range the following expression:

$$Z''_{r_n}(0) = \frac{2}{\upsilon^2_{gr}} \int_{-T/2}^{T/2} AA''\,dt - 2\int_{-T/2}^{T/2} k^2_e A^2\,dt +$$

$$+\frac{1}{E v_{gr}^{2}}\left(\int_{-T/2}^{T/2} t\varphi' A^{2} dt\right)^{2}.$$

The last two terms of this formula describe information obtained as a result of amplitude and phase modulation, the first relfecting data on velocity due to the Doppler shift of oscillation carrier frequency. It is easy to detect that, all other things being equal, the first term greatly exceeds in magnitude the two others. Thus, most often the measurement of velocity is done at the oscillation carrier frequency, and in this case the maximum value of the second derivative of the ACF with respect to velocity is expressed by the formula [13]

$$Z_{v}''(0)=-2k^{2}\int_{-T/2}^{T/2} t^{2} A^{2} dt. \tag{4.1.6}$$

Let us no move to the calculation of a mixed second derivative of the ACF. Taking into account assumptions on the even parity of the function A(t) and the odd parity of the first derivative of the modulating function, for the maximum value of the second mixed derivative we find the equation

$$Z_{r_n v}''(0)=-\frac{4k}{v_{gr}}\int_{-T/2}^{T/2} t\varphi' A^{2} dt. \tag{4.1.7}$$

Therefore, we are convinced that formulas (4.1.5), (4.1.6), and (4.1.7) are derived from the common formula (3.3.22), if it is used to evaluate constant quantities -- initial range and velocity of objects. /85

4.2. Accuracy of Phase and Pulse Telemetry Methods

The second stage of analysis of accuracy should be dedicated to an examination of the features of phase and pulse telemetry methods which are related to a number of methods which have received the widest use in space measuring complexes. In phase measurements, information on parameters of motion is included in the phase shift of the envelope of the received signal with respect to the envelope of the reference oscillation. The pulse method of measurements is based on defining the time lag of the received pulse with respect to the emitted pulse. In implementing the phase method, the initial definition and stabilization of instrument lag is ensured, which is tantamount to defining and stabilizing the initial phase of the envelope. A similar operation is likewise performed in measurements with pulse methods.

Formulas for evaluation of potential accuracy of phase telemetry methods for defining the parameters of motion may easily be derived from general relationships (3.3.22) and (3.3.23). In this regard, we must take into account that phase telemetry methods are mainly implemented at modulation frequencies and consequently, for measurements we use signals whose amplitude fluctuates in time in conformity to the harmonic law

$$A(t) = A_m(1 + m\cos\Omega t). \tag{4.2.1}$$

For the sake of simplicity, we will assume that signal phase modulation is absent.

Information included in the phase of carrier oscillation is not usually used in phase telemetry systems, and thus in evaluating the accuracy of measurements it is sufficient to bear in mind only the second term of formula (3.3.22). As a result, we derive the following expression for the maximum value of the second derivative of the ACF:

$$Z''_{ij}(0) = -\frac{m^2\Omega^2}{v_{\text{gr}}^2}\int_V\int_T \frac{\partial r}{\partial q_i}\,\frac{\partial r}{\partial q_j}\,A_m^2\,dV\,dt. \tag{4.2.2}$$

If signal amplitude fluctuates little during a measurement session, the second derivative of the ACF is reduced to the form /86

$$Z''_{ij}(0) = -\frac{m^2\Omega^2 A_m^2}{v_{\text{gr}}^2}\int_V\int_T \frac{\partial r}{\partial q_i}\,\frac{\partial r}{\partial q_j}\,dV\,dt. \tag{4.2.3}$$

In its structure, the maximum value of the second derivative is similar to the expression for the maximum value of the second derivative of the ACF of an unmodulated carrier having a known initial phase

$$Z''_{ij}(0) = -2k^2\int_V\int_T \frac{\partial r}{\partial q_i}\,\frac{\partial r}{\partial q_j}\,A^2\,dV\,dt, \tag{4.2.4}$$

which is completely natural.

The last factor of expression (4.2.3) -- the space-time integral of the product of partial derivatives -- with an accuracy to the constant factors coincides with the expression for the coefficients of equations used to process results of telemetric measurements aimed at defining the parameters of motion.

Examination of the phase telemetry method thus permits us to show the meaning and function of partial derivatives of instantaneous range with respect to definable parameters, which appear in all terms, without exception, of the general formula for the maximum values of the second derivatives of the ACF. Partial derivatives in formulas (3.3.22) and (3.3.23) reflect the stage of optimum signal processing which corresponds to the stage of "secondary" processing of trajectory information. The purpose of the stage of "secondary" processing consists, as we know, of defining the parameters of motion with respect to range measurement results.

It should be stated, however, that (4.2.2) and not (4.2.3) is more similar in content to the formula of coefficients of normal equations. Indeed, beneath the summation sign (or integral) in the expression for coefficients of normal equations, in addition to the products of partial derivatives, we must represent the weight coefficients, whose magnitudes are inversely proportional to the dispersions of isolated measurements. The function of these weight coefficients in this case is filled by the factors A_m^2dt, which reflect the influence of errors of some imaginary measurements dt in duration. Hidden within these factors is also the relationship as a function of distance between SV and the point of observation. Indeed, signal amplitude at the point of reception A_m is associated by an inversely proportional relation- /87 ship with the distance to the SV. Consequently, the factors A_m^2 allow for the relationship between signal strength and range to the SV and indicate the influence of this relationship on the accuracy of defining the parameters of motion.

Therefore, formula (4.2.2) and formulas in section 3.3, in addition to everything else, define the choice of weight coefficients in optimum signal processing; these coefficients are directly proportional to signal strength at the point of reception and consequently, inversely proportional to the square of the instantaneous range between the SV and the point of observation.

It should be noted that in forming the weight coefficients, several other quantities participate which lie beyond the limits of formulas for secondary derivatives of the ACF. Participating in this process, in particular, is the quantity of spectral density of noise N_0, which with $Z''_{ij}(0)$ enters into the expression for elements of correlation matrices cited in sections 3.1 and 3.2.

On the other hand, the material in section 4.1 clearly shows that the maximum values of the secondary derivatives of the ACF depict the process of measurement of instantaneous range and velocity of objects.

It is therefore clear that the formulas for maximum values of the second derivatives of the ACF encompass the measurement process of topocentric SV coordinates (and likewise, their derivatives) and the process of processing these coordinates to define the parameters of motion.

Let us now examine expressions for the maximum values of the second derivatives of the ACF corresponding to the specific conditions of pulse measurements. Let us assume that the received signal is in the form of short pulses which occur with a certain periodicity during the measurement session of T duration. Let us state that the pulses are so brief that the partial derivatives of range with respect to the parameters of motion, within the limits of pulse activity, may be considered constants.

With the foregone assumptions, the partial derivatives of range may be removed beyond the integral signs in formula (3.3.23); these derivatives, generally speaking, are functions of time and the coordinates of the reception point and are related to the moments of activity of the corresponding pulses. Assuming, as before, that the envelopes of the pulses are even, and the derivatives of modulating functions are odd functions of time, for the maximum value of the second derivative of the ACF we derive the equation /88

$$Z''_{ij}(0)=\frac{2}{q^2_{gr}}\sum_{l}\sum_{k}\frac{\partial r_{kl}}{\partial q_i}\frac{\partial r_{kl}}{\partial q_j}\int\limits_{V_l}\int\limits_{\tau_k}|\bar{A}'|^2 dV\,dt. \qquad (4.2.5)$$

In this formula, the space-time integral with respect to the area of reception, corresponding to all antennas of the complex and the entire time cycle of measurements, is replaced by a double summation of integrals, each of which is calculated with respect to an individual antenna of the complex and discrete pulse.

The integrals entering into individual terms of the last formula are second time derivatives of the ACF of discrete pulse signals, received by different antennas of the complex:

$$4\pi^2\int\limits_{V_l}\int\limits_{\tau_k}|A'|^2\,dV\,dt=Z''_t(0). \qquad (4.2.6)$$

They describe the potential accuracy of measurements of the time lag of isolated pulse signals. In general, formula (4.2.5) also is analogous to the formula for coefficients of normal equations, which is used in processing results of telemetry measurements; the integrals of (4.2.6) act as weight coefficients in these formulas.

The particular cases of using the electromagnetic field which have been examined in this section are associated with the use of information supplied mainly by modulating oscillations. In the measurement process, however, information may also be used which is contained in the phase of carrier oscillations. Information on carrier phase is implemented, as we know, in Doppler and goniometer measurements. Let us first examine the problem of accuracy of Doppler methods of defining the parameters of motion.

4.3. The Potential Accuracy of the Doppler Method /88

At present, electromagnetic fields whose initial phase is random and constant within the limits of some interval (the noise correlation interval) are used in practice for measurements. /89 Subsequently, it is impossible to realize phase methods with a measurement at the carrier frequency by means of such signals. Nevertheless, in the ranges used for communications with space vehicles, these methods are not realizable for another reason: for their execution, practically insurmountable difficulties arise in eliminating the ambiguity of the measurements. Nonetheless, with sufficiently high frequency stability (this means sufficiently great length of the phase's noise correlation interval), signals with an unknown initial phase can be used for defining the parameters of motion. Measurements become possible if, having rejected the use of information included in the carrier's initial phase, information is used which is contained in an increment of the phase in a measurement interval or, which is in essence the same, the frequency of the carrier waves. The corresponding methods of measurement have been called Doppler methods.

Doppler methods have in recent years been significantly developed in communications with progress in the areas of aviation and space technology. Successes in stabilizing the frequency generations by means of which sounding and reference signals are formed have played a significant role in this.

The materials in Chapter 3 allow us to evaluate the potential accuracy of Doppler methods in the general case where not only the velocity parameters, but also the elements of the objects are included in the number of definable parameters of motion. Let us assume at first that the on-board transmitter of the SV emits unmodulated waves which, upon reception on Earth, are fed to the signal input of a quadratic correlometer (Fig. 4.1). Let the reference signal representing the model of the signal to be received also be formed on Earth according to a priori data. The initial phase of the reference signal obviously does not have a value; however, a phase increment during the time of measurement and the temporary motion of measuring this increment, i.e., the frequency of the signal, should be matched so that they correspond to the phase increment and the frequency of the useful signal. Such congruence, as is well known, will be attained with congruence of the real and a priori values of the parameters of motion. Let us again note that, in contrast to phase range-measuring methods, with a measurement at the carrier frequency, information about the parameters of motion in Doppler measurements is derived from phase measurements and signal frequency measurements. Since the phase increments and frequency increments which we are discussing are due to the Doppler effect, the method described /90 is called the Doppler method.

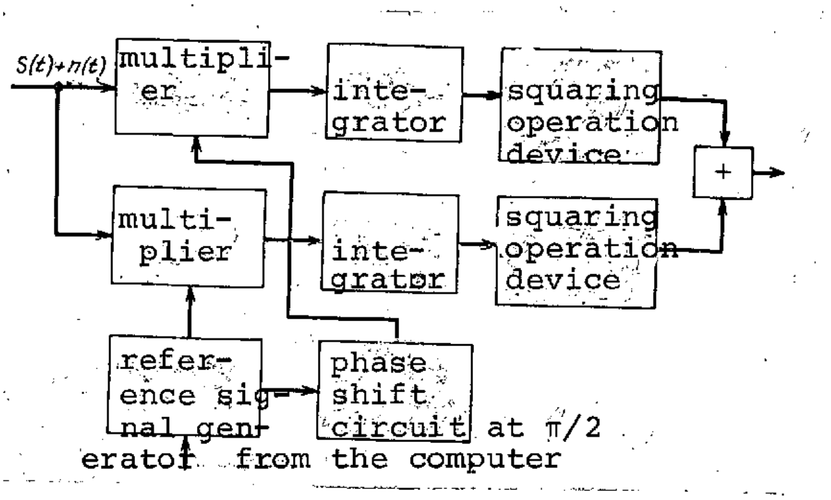

Fig. 4.1. Structural diagram of a square correlometer.

It follows from the arguments given that the last two terms of formula (3.3.23) give the overall idea of the potential accuracy of the Doppler method, since they reflect information contained in the carrier wave phase in conditions where the initial phase of this wave is not known and can assume any value within the limits of 0 to 2 π with equal validity. Thus, disregarding the "amplitude" terms of formula (3.3.23), we find that the potential accuracy of Doppler measurements is characterized by the following magnitude of the maximum value of the ACF second derivative with respect to the definable parameters:

$$Z''_{ij}(0) = -\frac{1}{2}\int\limits_V\int\limits_T \frac{\partial r}{\partial q_i}\,\frac{\partial r}{\partial q_j}\,k_e^2 A^2\,dV\,dt + \\ +\frac{1}{4E}\left(\int\limits_V\int\limits_T \frac{\partial r}{\partial q_i} k_e A^2\,dV\,dt\right)\left(\int\limits_V\int\limits_T \frac{\partial r}{\partial q_j} k_e A^2\,dV\,dt\right). \tag{4.3.1}$$

In deriving this formula, it was taken into account that Doppler systems use a predominantly non-inquiry method of operation.

Which parameters of motion can be determined by the Doppler method? It is evident that if the SV moved with constant velocity relative to the observer (i.e., only moved away from or closer to it), then it would be possible to measure only its velocity. Formula (4.3.1) verifies this. Actually, in the absence of phase modulation, the maximum value of the ACF second derivative with respect to the initial distance is equal to zero, although the maximum value of the ACF second derivative with respect to the velocity has a defined finite value. /91

It is easily noted that the ACF second derivative with respect to the initial distance becomes equal to zero because $dr/dr_n = 1$. However, in the overwhelming majority of actual situations, partial derivatives of the instantaneous range with respect to the SV's initial coordinates are different from 1, and equalization of the first term of formula (4.3.1) by the second, generally speaking, does not occur. Consequently, in these situations the ACF second derivative with respect to the coordinates will not be zero, which attests to the possibility of defining the initial conditions mentioned.

In shifting from an unmodulated signal to a phase-modulated signal and similarly disregarding previous information which was included in the signal's amplitude, we find that this transition does not lead to a significant change in the processes which occur in Doppler measurements, although phase modulation can lead to an increase or decrease in accuracy. As an examination of the first and second components of formula (4.3.1) shows, the potential accuracy is determined in a given case by the value of the effective wave number equal to $k_e = k + \phi'/\nu_{gr}$, and consequently if $\phi' > 0$, then $k_e > k$, and the accuracy of measurements by means of a phase-modulated signal will be greater than the accuracy of measurements on unmodulated carriers. It is interesting that an increase in the accuracy of Doppler measurements because of phase modulation is taken into account by the same terms of formula (3.3.22) which describe the increase in accuracy of pulsing methods of range measurements.

A model of a uniformly withdrawing or uniformly nearing object shows that measurement conditions exist which are unfavorable for application of the Doppler method. It is therefore advisable to examine the question of feasibility conditions for Doppler methods in a somewhat more general form. It is evident that the magnitude of the ACF second derivative, meaning, the accuracy of Doppler measurements, is greater than in other similar conditions the smaller the second term of formula (4.3.1) is in comparison with its first member. Accuracy completely depends on the magnitude of the difference between these terms. Consequently, for evaluating the conditions for

for attaining the greatest accuracy, it is necessary to develop conditions in which the difference referred to is maximum. /92

Using the Bunjakovskii-Schwartz inequality, restricted to the case of a signal with constant amplitude, for the diagonal elements of the ACF second derivatives, we will derive the following relation:

$$VT\int_V\int_T\left(\frac{\partial r}{\partial q_i}\right)^2 k_e^2 dV\,dt \geqslant \left(\int_V\int_T\frac{\partial r}{\partial q_i}k_e dV\,dt\right)^2. \qquad (4.3.2)$$

The equality in this formula is attained only in the case of the independence of the magnitude

$$\frac{\partial r}{\partial q_i}k_e \qquad (4.3.3)$$

from variable integrations, i.e., from time and the spatial position of the observation point. In the general case, the magnitude of the product (4.3.3) is a function of time and the coordinates of the reception point; consequently, the maximum value of the ACF second derivative will not be zero, i.e., the Doppler measurements will yield specific metric information.

Inequality (4.3.2) is increased with a stronger degree of variability of function (4.3.3) in the measurement interval, and it will be especially large if function (4.3.3) in the space-time area of reception is alternating. Finally, the inequality attains its ultimate value when the first part of inequality (4.3.2) becomes equal to zero. This occurs when function (4.3.3) is an odd function of coordinates and time. In the latter case, the potential accuracy of the Doppler method will be determined by the magnitude of the first component of formula (4.3.1) and, consequently, will be equal to the potential accuracy of the phase method of range measurements at the frequency of carrier fluctuations, i.e., the frequency obtained with a known initial phase of carrier fluctuations.

Only partial derivatives with respect to some parameters of motion can satisfy the condition of oddness. Parameters of motion whose partial derivatives do not satisfy this condition will be defined with less accuracy. Parameters whose partial derivatives are not functions of the coordinates and of time are generally not determined by Doppler measurements.

Thus, the potential accuracy of the Doppler method for defining parameters of motion is described by the maximum value of the ACF second derivative, represented by formula (4.3.1). /93

Let us return to this formula once more and turn our attention to some of its properties. First of all, it is significant that a wave number appears in this formula, meaning that the accuracy of measurements is determined by the frequency of the carrier fluctuations. This is a very important fact. Let us recall that the accuracy of phase range-measuring methods is a function of the frequency of the modulating fluctuations or the frequency of the pulses.

In further analyzing (4.3.1), it is impossible not to turn our attention to its connection with the formula for coefficients of systems of normal equations. It is seen that the first, fundamental term of formula (4.3.1) is analogous in structure to the formula for the coefficients mentioned. However, the analogy ends here, since with further examination, substantial differences are revealed between the formulas. The first is connected with the presence in (4.3.1) of a second component which is absent in the formula for coefficients of normal equations. The derivation and role of this component has already been discussed.

The second difference between the formulas is more important in a principal respect than the first; this is the difference in the composition of the first component's terms. In the formulas for coefficients of normal equations, formed in processing the results of Doppler measurements, the partial derivatives of the radial velocity component according to definable parameters [1, 3, 4] occur, and in (4.3.1), partial derivatives of the instantaneous range are shown instead of these. In this respect, it is clear that each of the terms of formula (4.3.1) separately or their algebraic sum do not result in the formulas for coefficients of normal equations with the partial derivatives of the radial velocity component. Thus, generally speaking, there is a definite difference between evaluating the accuracy of Doppler methods with frequency measurement data processing according to the method of least squares and evaluating the potential accuracy. Determining the degree of difference in accuracy evaluations is difficult in a general form; therefore, this question is not examined here. We will limit ourselves to one short remark.

We must keep in mind that the realization of a measurement procedure which ensures attainment of accuracy of equal potential is connected with surmounting significant difficulties and requires much more complex and expensive equipment than the execution of less precise procedures. In this connection, measurements are usually made in practice by recording the Doppler frequency shift or integrals of it after a fixed segment of time with subsequent processing of measurement data according to the method of least squares. This methodology is distinguished by its simplicity and very high efficiency. Only in those cases where especially high accuracy and resolution are required is correlation, i.e., optimal signal processing, used. Similar processing is used as necessary in radar systems for lateral scanning of the Earth's surface [27], which is an unusual ("non-space") example of the execution of a Doppler measurement method. /94

Moreover, it should be kept in mind that the optimization of measuring systems in practice is usually carried out not according to one, but according to several criteria, and in a number of cases not precision, but some other criterion plays a decisive role. This must also be taken into consideration in using the materials cited here.

§5.4, in which the evaluation of the potential accuracy of Doppler and range-measuring methods of SV measurements on one pass through the visibility range is given, plays the role of a model which illustrates the fundamental statements of the given section.

4.4. The Potential Accuracy of Azimuth Scale-Range Measurements

Let us apply the relations obtained in Chapter 3 to the particular case of azimuth scale-range measuring systems. A system whose antenna device dimensions are small in comparison with the distance to the SV is usually called an azimuth-scale system. If the antenna device consists of several spaced antennas, then not only the dimensions of the individual antennas, but also the distance between them must satisfy the condition mentioned. Because of the relative smallness of the antenna system, the directions to the SV from its different points can be considered parallel, and the problem of determining the spatial location of the SV is reduced to determining the distance to the source and two of its angular coordinates. If the observer has sufficiently precise a priori data available, then it is clear that in the measuring process, instead of defining the object's coordinates, we will be limited to a location having a comparatively small quantity of corrections to the a priori values of distance and angular coordinates.

Let us set up the problem of evaluating the potential ac- /95 curacy of azimuth scale-range-measuring systems, i.e., the maximum accuracy which can be obtained by means of an electromagnetic signal field which is to be recorded in a small area of space in comparison with the distance to the SV. We will assume that for the measurements, a field modulated by amplitude and phase whose initial phase is unknown is used. For simplification of the problem, we will also assume that the antenna has axis symmetry in relation to the direction to the field's source.

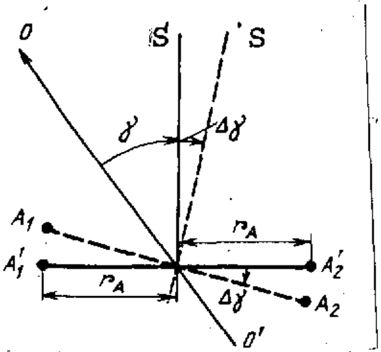

Fig. 4.2. The geometric relations in defining the SV's angular coordinates.

For the symmetry of an antenna, it is sufficient to evaluate the accuracy of defining only one complete angular coordinate.

We will derive an expression for the ACF signal second derivative, assuming that only two symmetrically positioned elements not having directivity are included in the antenna. We will use Fig. 4.2 for this, in which points S and S' are shown as the reference (a priori) and actual location of the source; points A_1 and A_2 are the reference positions of the antenna elements; A_1' and A_2' are the positions in which they appear after completing the process of taking a bearing; and r_A is the distance from the antenna elements to its axis. The letter γ designates the angular coordinate of the source; $\Delta\gamma$ is the difference between the a priori and actual values of the angles; and OO' is the origin of the angular coordinates.

We can see from the figure that the differences of a priori and true values of distances from the first and second antenna elements to the SV can be represented by the formulas

$$\left.\begin{aligned}\Delta r_1 &= \Delta r_0 + r_A\,\Delta\gamma,\\ \Delta r_2 &= \Delta r_0 - r_A\,\Delta\gamma,\end{aligned}\right\} \qquad (4.4.1)$$

where Δr_0 is the difference between the real and a priori distances from the center of the antenna to the SV.

The magnitudes Δr_0 and $\Delta\gamma$ are functions of the definable corrections to the parameters of motion; therefore, using a Taylor expansion, we obtain

$$\left.\begin{aligned}\Delta r_1 &= \left(\frac{\partial r_0}{\partial q_i} + r_A \frac{\partial \gamma}{\partial q_i}\right)\Delta q_i,\\ \Delta r_2 &= \left(\frac{\partial r_0}{\partial q_i} - r_A \frac{\partial \gamma}{\partial q_i}\right)\Delta q_i.\end{aligned}\right\} \tag{4.4.2}$$ /96

Consequently, in range-azimuth scale measurements, partial derivatives of the distance between the corresponding elements of the antenna field and the SV are equal to the sum or difference of two values:

$$\frac{\partial r_0}{\partial q_i} \mp r_A \frac{\partial \gamma}{\partial q_i}, \tag{4.4.3}$$

one of which is the partial derivative of the a priori value of the distance between the center of the antenna and the SV, and the other is proportional to the partial derivative of the a priori value of the angular coordinate.

Placing these values in (3.3.23) and taking into consideration that the ACF second derivative which describes the measurement process as a whole is equal to the sum of second derivatives calculated for the different antenna elements, we will obtain

$$\begin{aligned} Z''_{ij}(0) = {} & \frac{2}{v^2_{\text{gr}}}\int\limits_T \frac{\partial r_0}{\partial q_i}\,\frac{\partial r_0}{\partial q_j}\,AA''\,dt + \\ & + \frac{r_A^2}{v^2_{\text{gr}}}\int\limits_T \frac{\partial \gamma}{\partial q_i}\,\frac{\partial \gamma}{\partial q_j}\,AA''\,dt - \\ & -2\int\limits_T k^2_{\text{e}}\frac{\partial r_0}{\partial q_i}\,\frac{\partial r_0}{\partial q_j}\,A^2\,dt - r_A^2\int\limits_T k^2_{\text{e}}\frac{\partial \gamma}{\partial q_i}\,\frac{\partial \gamma}{\partial q_j}\,A^2\,dt + \\ & + \frac{1}{\text{E}}\left(\int\limits_T \frac{\partial r_0}{\partial q_i}\,k_{\text{e}}\,A^2\,dt\right)\left(\int\limits_T \frac{\partial r_0}{\partial q_j}\,k_{\text{e}}\,A^2\,dt\right); \end{aligned} \tag{4.4.4}$$

In this formula the case is represented where, for range measurements according to a retransmitted signal, the inquiry signal of a terrestial transmitter passes the doubled distance to the SV at the same time as in measuring the angles the output effect is determined by the quantity of ordinary differences of the distances from the individual antenna elements to the SV.

It is evident that in the transition from a two-element to a multi-element antenna, we will obtain a formula of the same structure. In the case of a multi-element interspaced antenna operation, the spatial integration obtained by deriving formula (4.4.4) must be completed, which for a linear antenna /97
reduces to integration with coordinate K_A. Moreover, instead of the power of a signal proportional to the square of the amplitude, it is necessary to examine the power of a signal coming to a unit of the cross-section or length of the antenna.

For example, for receiving signals on a flat antenna with a square aperture, the length of a side of which is equal to D, for a more informative, third term of formula (IV.4.4), we will derive the following expression

$$Z''_{ij}(0)=\frac{D^4}{8}\int_T k_e^2\frac{\partial\gamma}{\partial q_i}\,\frac{\partial\gamma}{\partial q_j}A_S^2\,dt. \tag{4.4.5}$$

Here A_S^2 is the flux density of the signal power.

In the case of reception on a circular antenna with diameter D_0, the third term of formula (4.4.4) assumes the form

$$Z''_{ij}(0)=\frac{\pi}{128}D_0^4\int_T k_e^2\frac{\partial\gamma}{\partial q_i}\,\frac{\partial\gamma}{\partial q_j}A_S^2\,dt. \tag{4.4.6}$$

These examples show that in passing from a two-element antenna to a single-element antenna, the composition and logical value of the terms of formula (4.4.4) do not change, and it is therefore possible to limit our examination only to this formula.

The first term of formula (4.4.4) describes the potential accuracy of measuring the distance from the center of the antenna to the SV by using information contained in the envelope; the third and fifth terms reflect the potential accuracy of Doppler measurements, and the second and fourth terms consider the potential accuracy of azimuth scale measurements (the second term corresponds to measurements with respect to the envelope, the fourth term with respect to the carrier fluctuation phase).

Which deductions allow analyzing the formulas obtained?

First of all, we can see that in receiving signals on a comparatively small antenna, the angular coordinates and the distance from the center of the antenna supply all the information about the parameters of motion which can generally be obtained by means of an electromagnetic field. As is to be expected, in measuring the distance from the center to the SV, azimuth scale measurements are totally equivalent to measurements of the distances from each point of the antenna to the SV. However, it can be seen from this that azimuth scale measurements without measuring the length from the center of the antenna to the SV allow using only a part of the informational possibilities of the field. /98

It is further possible to conclude that all the components of formula (4.4.4), responsible for the accuracy of range and azimuth scale measurements, are analogous in structure to the formulas for coefficients of normal equations used in processing range-measuring and azimuth scale data.

We can also see from formula (4.4.4) that the absence of data about the initial phase of the electromagnetic field does not have an influence on the potential accuracy of azimuth scale measurements. Finally, a characteristic feature of this formula which must be acknowledged is the absence of terms representing information about the velocity of the angular shift of the objects. As was already noted in the analysis of formulas (3.3.22) and (3.3.23), this does not mean that the similar information of formula (4.4.4) is generally not taken into consideration -- it is implicitly considered.

4.5. A Model for Realizing the Principle of Optimal Signal Filtration: The Planetary Radar of the Academy of Sciences of the U.S.S.R.

In concluding the description of the potential possibilities of different methods for defining the parameters of motion, we will give a model of a system of orbital measurements in which the principles of optimal filtration, described in this book, are basically realized. Planetary radar [11] serves as the model of such a system.

The basic purpose of radar consisted of making the absolute magnitude of an astronomical unit more precise -- the important constant which is included in the equation for the motion of the Earth and other planets of the solar system in the form of a unique parameter of motion.

The general concept of defining an astronomical unit by means of radar was included in the selection of a value for this magnitude in which the calculated values of the instantaneous phase lags and Doppler frequencies of signals deflected from the planets were equal to the measured values of these magnitudes. A phase method of range measurements on frequency modulation and the Doppler method of measuring radial velocity components at the carrier fluctuation frequency were used in radar. Amplitude manipulating fluctuations whose frequencies were distinguished by high stability (10^{-9}) served as the signal. The method of frequency manipulation was also used, but it was of secondary value. The manipulation frequencies were close to 4 and 8 Hz.

The magnitude of the correction to the astronomical unit was judged according to the output effects of a correlometer and narrow-band filters; a correlometer was used for discriminating the envelope of the amplitude manipulating signal and filters at the output of the last frequency conversion circuit were used for discriminating the carrier.

Methods of receiving and discriminating signals used in radar have a number of characteristics which are technical in nature.

1. The predicted values of the envelope phase lag and Doppler shifts of the carrier and envelope fluctuations were not fed into the receiving side of the system (for forming a correlator reference signal), but into the transmitting side -- for forming a transmitting (sounding) signal. /99

Due to this, selective fixed-tuning filters were successfully used for discriminating carrier fluctuations. The frequencies of the tuning filters overlapped the range of expected values for signal frequencies after the last frequency conversion. By means of the filters, the magnitude of the Doppler frequency shift of the signals received was determined.

2. The possibility of determining and recording the correlation function of the amplitude manipulating signal's envelope was provided for in radar. According to the value of the reference signal phase shift with respect to the sounding signal in which the output signal of the correlometer attains the maximum, a deduction can be made about the difference between the real and predicted values of the received signal's phase lag with respect to the sounding signal.

3. The received signal's envelope lag with respect to the sounding signal is determined in a sequential method by means of repeated reproduction of the receiver's output signal which was recorded with different values of the reference signal lag.

Due to storing the received signal, more efficient use of signal energy at observation was obtained (the lengthy process of selecting a priori data values closer to the real values was done after the end of the communication session), and excessive complication of the analyzer circuit, in which a parallel optimal filtration circuit was used, was avoided.

A great deal of more interesting information can be obtained from examining the functional circuitry of planetary radar. Radar can be divided into three basic component parts. The first is properly radar with a signal recording system which acts on the output of the last frequency conversion circuit. A simplified structural diagram of this part is shown in Fig. 4.3. The second part of radar includes two weak signal analyzers. A simplified structural diagram of one of the analyzers is presented in Fig. 4.4. A signal recording /100

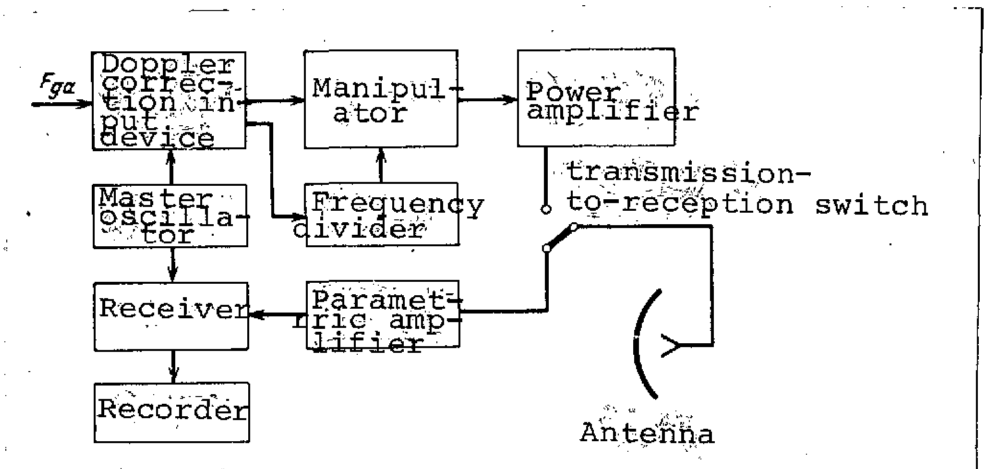

Fig. 3.3. Structural diagram of a planetary radar receiving-transmitting device.

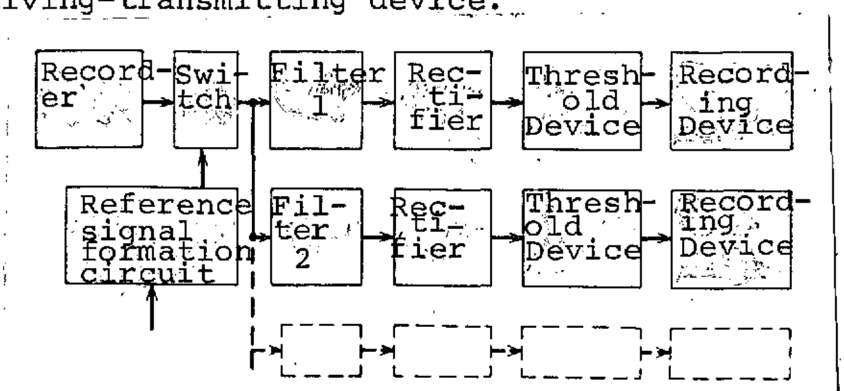

Fig. 4.4. Structural diagram of a planetary radar weak signal analyzer.

system, for which a tape recorder is used, is the connecting link between both parts.

The third component part is a computer, by means of which the calculation for predicting the distance and Doppler correction and for defining the astronomical unit is carried out.

The functional circuitry characteristic of radar does not require any elucidation; therefore, commentary will only concern the analyzer circuitry (Fig. 4.4). There is a switch at the analyzer's input which controls the reference fluctuations of the manipulation frequency. By means of this switch, multiplication operations of the received signal's envelope and the reference fluctuation are carried out, composing the first stage of defining the correlation function. Narrow-band filters, linear detectors, threshold circuits and recording devices which simultaneously serve as correlometer integrators, come after the switch. The duration of integration is 5 minutes. Signal recording and storage, effective within the limits of the first and second half-periods of the reference signal, are carried out separately. The difference in the values of the output voltages with respect to the first and second half-periods defines one value of the correlation function. Thus, a number of correlation function values are recorded, and the reference signal lag, in which the correlation function attains its maximum, is defined.

By means of planetary radar, the accuracy of defining an astronomical unit was increased by more than two orders of magnitude. Radar also allowed the attainment of much more important scientific information about the planets of the solar system.

Chapter 5 /101

POTENTIAL ACCURACY OF DEFINING DIFFERENT SYSTEMS OF PARAMETERS OF MOTION

5.1. Content of the Problem

As is known, various geometric and kinematic values which uniquely describe the law of motion of a SV (or the position of the observer) appear as definable parameters of motion. Derivatives of distance according to the definable parameters of motion which depend on the choice of these parameters and on the coordinate system in which they occur are part of subintegral expressions of autocorrelation function second derivatives. Consequently, the potential accuracy of the complexes will be a function of these factors.

An investigation of this dependency and the selection of coordinate systems which ensure high accuracy of definitions or a more adequate reflection of the possibilities of measurement complexes, comprise the basic content of the present chapter.

Before beginning to examine the questions mentioned, it is appropriate to consider the physical interpretation of the orbital or navigational (geodetic) measurement process.

The physical picture of the phenomena which take place in defining the parameters of the SV's motion or in measuring navigational and geodetic parameters is quite evident. It can be represented in the following manner.

In carrying out measurements by means of a given radio-engineering system during one pass of an artificial earth satellite in the visibility range, we will obtain a set of the position's planes. In the case of range measurements which describe the potential accuracy of determining the SV's parameters of motion, a set of concentric spherical planes whose center, in observation from Earth, is placed at the point of the observer's location, is obtained. A fixed set of values for the received signals' parameters correspond to this set of positional planes. The measurement process consists of comparing these signal parameter values with the corresponding a priori reference signal parameter values which are constructed by means of a priori information about the SV's movement with respect to the observer. The problem consists of the fact that judgments must be made about the difference between the a priori and real values of the parameters of motion and about the /102 actual law of motion of the SV with respect to the differences between the a priori and real values of the signal parameters.

It is evident that the parameters of motion will be defined more accurately with greater correspondence of the signal parameter deviations to the given deviations of the parameters of motion from their nominal values. In turn, this will occur first of all in the case where the gradients of the location's planes are sufficiently great. However, the problem is not exhausted by the dependency on the magnitude of the gradients, as this occurs in normal position-finding. If we solve the navigational problem by defining the position of an airplane or ship, then it is necessary that the angles of intersection of the different position planes fully satisfy the defined conditions.

Analogous conditions must be fulfilled in defining the spatial position of the SV, although at first glance, the statement about the angles of intersection of the SV's position planes, whose location changes from measurement to measurement, does not seem fruitful. In reality, in conditions where there is sufficiently extensive a priori information, it is possible to use the assertion about the position plane angles of intersection, and this offers the possibility of achieving the conversion from non-simultaneous measurements of the same geometric magnitude to defining the SV's position in two- and three-dimensional space. Due to the use of a priori data, the results of non-simultaneous measurements can be reduced to one point in space, and parallelly transferred a priori and real position planes obtained in the process of measurements offer the possibility of judging the magnitude of the deviations between the a priori and actual values of the parameters of motion. The principle of the SV's movement with respect to these data can be reproduced more precisely with more favorable angles of intersection between the position's planes, i.e., the greater the changes are which the directions normal to the position plane at the time of measurement undergo.

Finally, it was shown that accuracy can also be a function of the choice of a system of coordinates in which the measurement results are represented. The essence of the given question is contained in the following.

As experience shows, together with the coordinate systems in which linear values and their derivatives are used for expressing the SV's position and velocity, coordinate systems are used in practice in which the SV's position and velocity /103
are expressed by means of a combination of linear and angular magnitudes and their derivatives (a spherical system of co-

ordinates) or by means of a combination of three angles, time, linear and dimensionless magnitudes (Keplerian elements), etc. In this connection, the dimensionality of the discrete elements of the system of coordinates used cannot coincide with the dimensionality of the electromagnetic field parameter in which information about the motion is contained. In the most general case, the distance, i.e., a linear value, appears, as we know, as such a parameter. Therefore, in the composition of analytic expressions which describe the accuracy of defining parameters of motion in a system whose coordinates are heterogeneous from the point of view of dimensionality with lengths or their derivatives, it is necessary to consider the coefficients in these expressions which describe the relationship of the dimensions of defined and initial magnitudes.

On the other hand, in examining the properties of coordinate systems used for representing the final measurement results, we will encounter a dependency of the defined parameters' errors on the magnitudes of these parameters. Errors in defining the coordinates and velocity of objects in rectangular Cartesian coordinate systems for selected units of measurements are not functions of the position and velocity of the SV's movement, and the accuracy of defining parameters of motion with known units of measurements do not depend on the choice of coordinate systems. However, if angular coordinates are used for representing the SV's position, and angular errors in defining the coordinates are used instead of linear errors, then it is obvious that the errors in defining the angles and angular velocities corresponding to the values of the linear errors in defining the spatial position and velocity of the SV will be a function of the position of the SV relative to the origin of the coordinates, although it is clear that in the reverse transition to linear errors, we will naturally eliminate such a dependence.

Such are the initial physical considerations which emerge when we begin analyzing the potential accuracy of a complex which measures the SV's parameters of motion.

The complete quantitative characteristics of all the phenomena mentioned are given by a matrix of second derivatives of the signal field autocorrelation function according to the SV's definable parameters of motion. We will attempt to explain how these phenomena are quantitatively reflected in formulas for the second derivatives of the autocorrelation function, and that there is no possibility of breaking these formulas down into component parts corresponding to the different stages of the measurement process.

5.2. The Dependency of Potential Accuracy on the Selection of Systems of Defined Parameters /104

Turning to the formula for the second derivative of the autocorrelation function according to definable parameters of motion

$$
\begin{aligned}
Z''_{ij}(0) = & -\frac{1}{v_{gr}} \int_V \int_T \frac{\partial^2 r}{\partial q_i \partial q_j} AA' \, dV \, dt + \\
& + 2 \int_V \int_T \frac{\partial r}{\partial q_i} \frac{\partial r}{\partial q_j} \left(\frac{1}{v_{gr}^2} AA'' - k_e^2 A^2 \right) dV \, dt + \\
& + \frac{1}{E} \left(\int_V \int_T \frac{\partial r}{\partial q_i} k_e A^2 \, dV \, dt \right) \times \\
& \times \left(\int_V \int_T \frac{\partial r}{\partial q_j} k_e A^2 \, dV \, dt \right),
\end{aligned}
\tag{5.2.1}
$$

where $k_e = k + \phi'/v_{gr}$, we will recall that the fundamental difference of this formula and formulas for evaluating the potential accuracy of defining primary parameters lies in the fact that partial derivatives differing by a unit from the instantaneous range between the SV and the observer with respect to the defined parameters of motion are included in it.

Derivatives of the instantaneous range according to the definable parameters of motion can be expressed by derivatives of the instantaneous range according to the coordinate components of velocity with respect to some fixed moment of time, and the derivatives of the coordinates and components of velocity according to definable parameters of motion with respect to the same moment of time.

Geocentric, topocentric rectangular, spherical or cylindrical systems can be used as coordinate systems in which the initial conditions are fixed. For determinancy, we will assume that a rectangular topocentric system of coordinates ξ, η, ζ is used, which can be inertial, Greenwich, meridional, or any other in their orientation.

In designating the derivatives of initial coordinates ξ, η, ζ in time by these same letters with points ξ, η, ζ, and using a generalized notation for the initial values of the coordinates /105
and velocities in a moment of time, to which are related

the results of defining the parameters of motion $\xi=\xi_1$, $\eta=\xi_2$, ..., $\zeta=\xi_6$, we will obtain, for the partial derivative of the range with respect to definable parameters of motion, the following relation:

$$\frac{\partial r}{\partial q_i}=\sum_{k=1}^{6}\frac{\partial r}{\partial \xi_k}\frac{\partial \xi_k}{\partial q_i}, \quad i=1, 2, \ldots, 6. \tag{5.2.2}$$

Consequently, the products of the partial derivatives will be expressed by the functions

$$\frac{\partial r}{\partial q_i}\frac{\partial r}{\partial q_j}=\sum_{k=1}^{6}\sum_{l=1}^{6}\frac{\partial r}{\partial \xi_k}\frac{\partial r}{\partial \xi_l}\frac{\partial \xi_k}{\partial q_i}\frac{\partial \xi_l}{\partial q_j}. \tag{5.2.3}$$

The second derivatives of the range according to the definable parameters of motion can be represented by means of the following formulas:

$$\frac{\partial^2 r}{\partial q_i \partial q_j}=\sum_{k=1}^{6}\sum_{l=1}^{6}\frac{\partial^2 r}{\partial \xi_k \partial \xi_l}\frac{\partial \xi_k}{\partial q_i}\frac{\partial \xi_l}{\partial q_j}+\sum_{k=1}^{6}\frac{\partial r}{\partial \xi_k}\frac{\partial^2 \xi_k}{\partial q_i \partial q_j}. \tag{5.2.4}$$

Derivatives $\delta\xi_k/\delta q_i$ are not functions of time and spatial variables by which integration into formula (5.2.1) is effected. Therefore, the expression for the second derivative of the autocorrelation function can be written in the form

$$\begin{aligned} Z''_{ij}(0)=\sum_{k=1}^{6}\sum_{l=1}^{6}\frac{\partial \xi_k}{\partial q_i}\frac{\partial \xi_l}{\partial q_j}\Bigg[\frac{1}{v_{gr}}\int_V\int_T\frac{\partial^2 r}{\partial \xi_k \partial \xi_l}AA'\,dV\,dt+ \\ +2\int_V\int_T\frac{\partial r}{\partial \xi_k}\frac{\partial r}{\partial \xi_l}\left(\frac{1}{v^2_{gr}}AA''-k^2_e A^2\right)dV\,dt+ \\ +\frac{1}{E}\left(\int_V\int_T\frac{\partial r}{\partial \xi_k}A^2 k_e\,dV\,dt\right)\left(\int_V\int_T\frac{\partial r}{\partial \xi_l}A^2 k_e\,dV\,dt\right)\Bigg]- \\ -\sum_{k=1}^{6}\frac{\partial^2 \xi_k}{\partial q_i \partial q_j}\frac{1}{v_{gr}}\int_V\int_T\frac{\partial r}{\partial \xi_k}AA'\,dV\,dt. \end{aligned} \tag{5.2.5}$$

We should note that the terms in the subintegral expressions which represent the first derivatives of the signal amplitude usually approach zero, since the signals actually used, as a rule, have a symmetrical form. Taking this into account, the formulas for the second derivatives of the ACF acquire the following form: /106

$$Z''_{ij}(0)=2\sum_{k=1}^{6}\sum_{l=1}^{6}\frac{\partial\xi_k}{\partial q_i}\frac{\partial\xi_l}{\partial q_j}\left[\int_V\int_T\frac{\partial r}{\partial\xi_k}\frac{\partial r}{\partial\xi_l}\times\right.$$
$$\times\left(\frac{1}{v^2_{gr}}AA''-k^2_e A^2\right)dV\,dt+\frac{1}{E}\left(\int_V\int_T\frac{\partial r}{\partial\xi_k}A^2 k_e\,dV\,dt\right)\times$$
$$\left.\times\left(\int_V\int_T\frac{\partial r}{\partial\xi_l}A^2 k_e\,dV\,dt\right)\right]. \tag{5.2.6}$$

Formula (5.2.6) is one of the elements of the ACF second derivatives matrix of maximum values. Using vector symbolics similar to what was done in §3.4, it is possible to write the entire set of elements for this matrix. For this, we must construct the matrix analogs of formulas (5.2.2) and (5.2.3). Forming the matrix analogs of these formulas is done in the following way. We insert a vector row of the partial derivatives of the instantaneous range to the SV according to the initial values of the coordinates and components of velocity:

$$\frac{\partial r}{\partial\xi}=\left\|\frac{\partial r}{\partial\xi_1}\;\frac{\partial r}{\partial\xi_2}\;\frac{\partial r}{\partial\xi_3}\;\frac{\partial r}{\partial\xi_4}\;\frac{\partial r}{\partial\xi_5}\;\frac{\partial r}{\partial\xi_6}\right\|. \tag{5.2.7}$$

We must emphasize that the letters $\xi_1 \div \xi_6$ designate the coordinates and components of the SV's velocity with respect to a fixed moment in time, to which the results of defining parameters of motion q are related. It is also important to keep in mind that no restrictions are placed on selecting the position of the origin and orientation of the axes of the coordinate system in which the initial values of the coordinates and velocity are represented. With respect to considerations discussed somewhat later, we will also assume that this is a system of Cartesian coordinates.

The vector row of partial derivatives of distance with respect to definable parameters of motion is connected to vector row (5.2.7) by the relation

$$\frac{\partial r}{\partial \mathbf{q}}=\frac{\partial r}{\partial \xi}\,\frac{\partial \xi}{\partial \mathbf{q}}=\frac{\partial r}{\partial \xi}\,\mathbf{J}_{\xi\,\mathbf{q}}, \tag{5.2.8}$$

where

$$\mathbf{J}_{\xi\,\mathbf{q}}=\frac{\partial \xi}{\partial \mathbf{q}}=\begin{Vmatrix} \frac{\partial \xi_1}{\partial q_1} & \frac{\partial \xi_1}{\partial q_2} & \frac{\partial \xi_1}{\partial q_3} & \frac{\partial \xi_1}{\partial q_4} & \frac{\partial q_1}{\partial q_5} & \frac{\partial \xi_1}{\partial q_6} \\ \frac{\partial \xi_2}{\partial q_1} & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial \xi_6}{\partial q_1} & \cdots & \cdots & \cdots & \cdots & \frac{\partial \xi_6}{\partial q_6} \end{Vmatrix} \tag{5.2.9}$$ /107

is the Jacobi matrix of transition from initial conditions ξ to parameters of motion $\mathbf{q}$ with respect to the moment of defining the parameters of motion.

The matrix of the products of the instantaneous range partial derivatives with respect to the definable parameters of motion assumes the form

$$\left\| \frac{\partial r}{\partial q_i}\;\frac{\partial r}{\partial q_j} \right\| = \left[\frac{\partial r}{\partial \mathbf{q}}\right]^{\mathrm{T}} \frac{\partial r}{\partial \mathbf{q}} = \mathbf{J}^{\mathrm{T}}_{\xi\mathbf{q}} \left[\frac{dr}{\partial \xi}\right]^{\mathrm{T}} \frac{\partial r}{\partial \xi}\,\mathbf{J}_{\xi\,\mathbf{q}}. \tag{5.2.10}$$

Consequently, the matrices of maximum values for the ACF second derivatives with respect to the definable parameters of motion and the initial conditions are interconnected by the relation

$$\mathbf{Z}''_{\mathbf{q}}(0)=\mathbf{J}^{\mathrm{T}}_{\xi\,\mathbf{q}}\;\mathbf{Z}''_{\xi}(0)\,\mathbf{J}_{\xi\,\mathbf{q}}, \tag{5.2.11}$$

where

$$\mathbf{Z}''_{\xi}(0)=2\int_V\int_T \left[\frac{\partial r}{\partial \xi}\right]^{\mathrm{T}} \frac{\partial r}{\partial \xi}\left(\frac{1}{v^2_{\mathrm{gr}}}AA''-k^2_{\mathrm{e}}A^2\right)dV\,dt+$$
$$+\frac{1}{\mathrm{E}}\left(\int_V\int_T\left[\frac{\partial r}{\partial \xi}\right]^{\mathrm{T}} k_{\mathrm{e}}A^2\,dV\,dt\right)\left(\int_V\int_T \frac{\partial r}{\partial \xi}k_{\mathrm{e}}A^2\,dV\,dt\right). \tag{5.2.12}$$

Finally, using the vector notation adopted in §3.4, it is also possible to represent the last formula in the following form:

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$$Z''_{\xi}(0) = 2\int\limits_{V}\int\limits_{T}\frac{1}{r^2}\left[\frac{\partial x}{\partial \xi}\right]^{T} x\, x^{T}\frac{\partial x}{\partial \xi}\left(\frac{1}{v^2_{gr}}AA'' - k^2_{e}A^2\right)dV\,dt + \frac{1}{E}\left(\int\limits_{V}\int\limits_{T}\frac{1}{r}\left[\frac{\partial x}{\partial \xi}\right]^{T} x\, k_{e}A^2\,dV\,dt\right)\times \times\left(\int\limits_{V}\int\limits_{T}\frac{1}{r}x^{T}\frac{\partial x}{\partial \xi}k_{e}A^2\,dV\,dt\right). \quad (5.2.13)$$

Here, X is the radius vector of the SV given in some topocentric system of coordinates.

Formulas (5.2.6) and (5.2.11) describe the potential accuracy of defining parameters q with measurements during time T by means of a three-dimensional antenna occupying area of space V.

These formulas are very interesting since they reflect the effect of the autocorrelation function on the second derivatives, and consequently on the accuracy of measuring two groups of factors which substantially differ according to content.

Factors which are a function of the properties and potentials of radio-engineering facilities and of measurement conditions are related to the first group. The second group consists of factors purely geometric in nature, connected with the properties of coordinate systems used for representing the measurement results.

The effect of the first group's factors is represented by quadruple integrals, calculated with respect to the area of space occupied by the receiving antennas during the measurements. These integrals take into account the intensity, frequency or width of the signals, the dimensions and position of the antenna systems, the form, length and position of the measured trajectory segment with respect to the ground facilities.

The integrals are also a function of the properties of the coordinate system $O\xi\eta\zeta$, by means of which the defined initial conditions are represented, and of the position of the origin and orientation of these coordinate axes. In this connection, measurement conditions are calculated by the partial derivatives which represent the projection of distance gradients on axis ξ, η, ζ at different moments of time and the magnitudes related to them -- the corresponding partial derivatives of the distance with respect to velocity.

The quantitative reflection of the effect of the second group's factors are included in the properties of the second derivatives of the initial conditions according to definable parameters of motion. These partial derivatives represent the connection between the initial conditions and the definable parameters of motion at the same moment of time and describe /109 only the process of coordinate transformations in the transition from a system of coordinates in which the initial conditions are given to a system of coordinates in which the definable parameters of motion are represented.

We can see from the formulas shown that the ACF signal field second derivatives with respect to defined parameters of motion, generally speaking, are functions not only of the properties of the electromagnetic field and the measurement conditions (i.e., of the form and relative position of the measured trajectory segment with respect to the antenna field), but also of the geometric properties of the magnitudes used for expressing the definable parameters of motion.

Not discussing here the details of the question of coordinate transformations, to which Chapter 6 is devoted, we will emphasize the apparent conditional character of this function. The reasons for the appearance of such a seemingly unusual function consist of the following.

The accuracy of defining an object's spatial position is described by the distance between two points, one of which corresponds to the real and the other to the erroneously found position. If geometrical magnitudes having such a property that the distance expressed by means of them is shown as a function of the object's position in space are used for representing this distance, then errors in defining the parameters of motion, meaning the signal field ACF second derivatives with respect to the parameters of motion represented by these geometric values, will also be a function of the parameters of motion used. And on the contrary, if magnitudes which allow expressing the distance by a relation which is not a function of these parameters are used as the parameters of motion, then a similar function will not be characteristic of errors in defining the parameters of motion.

This phenomenon in itself does not result in a decrease in accuracy to which the fact attests that, in the transition from parameters of motion in which the indicated property is inherent to a Cartesian system of coordinates lacking this property, the dependence of the errors on the position of the SV disappears. However, as we know, the SV's orbits are not always given by the initial conditions in Cartesian coordinates.

Moreover, separate inspection of all the mechanisms for the origin of errors is difficult, and only the final values of errors in defining the parameters of motion are usually of interest. In these conditions, an increase in the numerical values of the errors in defining some components of the position vector in the transition from one point in space to another /110 can be assigned due to the imperfection of the measuring systems. Therefore, experiments in perfecting a measuring complex, for example, by means of increasing the signal energy, can be undertaken. Meanwhile, it is obvious that similar efforts are appropriate only if it is certain that the undesirable dependence of the accuracy on the position of the SV is not connected with the properties of the coordinate systems. In order to find such a certainty, it is necessary to express the complex errors in linear values, i.e., to recalculate the accuracy evaluation results in a Cartesian system of coordinates, and only by analyzing the errors in this system is it possible to make a really correct judgment about the necessity of perfecting the complex's measuring agents or changing the position of its elements on the Earth's surface.

Such is the essence of the question concerning the choice of a system of definable parameters of motion.

5.3. Some Properties of Coordinate Transformations

Correlation matrices which describe the minimally attainable values of errors in defining parameters of motion are related to the maximum values of the ACF second derivatives by the formula

$$B^{-1} = B_a^{-1} - \frac{2}{N_0} \sum_k \frac{E_k}{N_0 + E_k} J^T_{\xi q} Z''_{\xi}(0) J_{\xi q}, \qquad (5.3.1)$$

which results from relations (3.2.15) and (5.2.11).

The last component of this formula is a matrix, an inverse correlation matrix of minimally attainable values for measurement errors B_m. Consequently,

$$B_m = -\frac{N_0}{2}\frac{1}{\sum_k E_k/(N_0+E_k)} J_{\xi q}^{-1}[Z_\xi''(0)]^{-1}[J_{\xi q}^{-1}]^T \qquad (5.3.2)$$

It is also possible to write the last matrix in the following manner:

$$B_m = -\frac{N_0}{2}\frac{1}{\sum_k E_k/(N_0+E_k)} \frac{[J_{\xi q}^T Z_\xi''(0) J_{\xi q}]^*}{(\det J_{\xi q})^2 \det Z_\xi''(0)} \qquad (5.3.3)$$

Here, the symbol det designates the determinant, and the sign * designates a matrix adjoined to the given matrix. /111

From the formulas derived, it is seen that the accuracy in defining the parameters of motion is a function both of the properties of the ACF second derivative matrix according to the topocentric coordinates, and also of the Jacobian of the properties of coordinate transformations leading from the topocentric coordinates to final values, by means of which the definable parameters of motion are described.

It also follows from this formula that in transforming the coordinates, the volume of the correlation ellipsoid, generally speaking, changes. The volume remains unchanged only for coordinate transformations for which the Jacobian of the transformation is equal to $\pm$ 1. Similar transformations, as we know, are executed by the transition from one orthogonal base to another, also orthogonal, base. However, in those cases where one of the bases -- the initial or final one -- is nonorthogonal, the correlation ellipsoid is deformed in the process of transforming the coordinates. Therefore, the Jacobian of the transformation can show a difference of one, either because of the difference in the physical dimensions of the initial and final coordinates, or because of the geometric pecularities of the coordinate systems, about which we have already spoken, and which consist of the fact that the same value of the linear error for the representation by means of these coordinates is shown as a function of the object's position. It is further obvious that at those points in which the Jacobian of the transformation has properties (approaching zero), the measurement errors increase, approaching infinity.

Finally, we can conclude from examining correlation matrix (5.3.3) that the matrix rank of the ACF second derivatives, along the vector of parameters q, does not exceed the rank of the Jacobian of the transformation matrix and the rank of the initial second derivative matrix. With an ordinary Jacobian of the transformation matrix, it will be equal to the rank of the ACF second derivative matrix according to the topocentric coordinates. Consequently, the dimensions of the vector of defined parameters of motion q (dimensions in the sense of the quantity of vector components) cannot exceed the dimensions of the topocentric coordinate vector ξ; this means that the determinant of the ACF second derivative matrix will be equal to zero with respect to defined parameters q if the dimensions of the topocentric coordinate vector and their derivatives ξ are less than the dimensions of the definable parameters vector q.

5.4. A Rough Estimate of the Potential Accuracy in Defining the Parameters of Motion of Narrow-Orbit SVs for Range and Doppler Method Measurements on One Pass in the Visibility Range /112

In this section, an example is given for using methods of evaluating the potential accuracy of measuring agents, discussed in the third and fourth chapters, for solving a practical problem. The materials in the section allow us to clearly demonstrate the basic properties and different aspects of range-measuring and Doppler methods and offer the possibility of examining the question concerning the informativeness of various segments of the SV's measured trajectory segments.

For solving the problem, the basic results of the fifth chapter are also taken into consideration: the definable parameters of motion are chosen in such a way that the undesirable effect of coordinate transformations are excluded. The rectangular coordinates of the SV at the moment of flight at the trajectory point least removed from the observer appear as the definable parameters of motion. This point is called the traverse of the observer.

The problem consists of evaluating the potential accuracy of the determinants of the indicated coordinates, i.e., calculating a maximum value matrix for the ACF second derivatives and a correlation matrix of minimally attainable error values.

Solving the given problem in a general form, i.e., with any principle of SV movement, is difficult. Therefore, we will limit ourselves to examining the simplest principle of the SV's movement which will allow obtaining results in a final form. We will assume that the satellite moves with constant velocity ν along a rectilinear trajectory and that the length of the trajectory segment to be measured is equal to $2\nu T$, where $2T$ is the total measurement time.

It should be noted that a linear approximation of the measured trajectory segment is not always permissible. It can be used if the altitude of the SV's trajectory over the Earth is comparatively low, since the energy conditions for low orbits are more favorable at the receiving point for a flight on the close traverse segment of a trajectory of comparatively small length, which can be approximated by a segment of a straight line. It is easy to calculate that the energy of the signals received from the SV, available in the visibility range, are some ten times less than the power of signals received from the SV at the moment of traverse flight. /113

Finally, we will assume that the signal phase fluctuation correlation is equal to the duration of the measurements and signal reception is done on a non-directional antenna.

The important question in the problem to be solved is that of selecting values which are considered invariable in the measuring conditions. Usually, such values are the power of the signal to be received and the spectral density of the noise at the radio receiver's input. Experience shows that such a choice of fixed values is more rational for estimating the signal's parameters. However, for solving the problem according to an estimate of the orbit parameters, or the coordinates of the observer, or other parameters together with those taken, fixing the signal's energy complicates the comparison of different methods of measurement, since the power of the received signal is a function not only of the transmission energy, the area of the receiving antenna and the duration of the measurements, but also of the orbit parameters and the observer's position with respect to the measured trajectory segment. Therefore, in the given problem, fixing the energy or power of the transmitting signal, the area of the receiving antenna and the spectral noise density are more advisable. In setting the energy characteristics of the emitted signal, we will obtain complete identity of the conditions in which the different measurements methods are compared.

In this connection, a separate examination of two cases is sensible: where the power of the emitted signal and the duration of the measurements (i.e., the energy of the emission is fixed) or only the power of the emission is fixed. Fixing the power of emission offers us the possibility of studying the potential possibilities of the measuring systems for measuring during the entire time of the SV's period in the visibility range and computing not only the negative, but also the positive results of an increase in the SV's flight altitude. Subsequently, an assumption about the constancy of the emission energy is essential. Turning to the duration of measurements on one pass to infinity, we will obtain the possibility of estimating the accuracy of measurements during the entire time of the SV's stay in the visibility range.

We will first assume that the phase method of measuring distance in frequency modulation is used, and that the information contained in the signal amplitude is not directly used.

We will limit our examination of the measuring process only to the most exact scale and will assume that elimination of ambiguities will be attained because of measurements on several "crude" scales and that, consequently, the total energy expended in the measuring process will several times /114
exceed the energy expenditures on an exact scale.

It is evident that the square of the signal amplitude at the receiving point is connected with the energy of emission P, antenna area S and instantaneous range r(t) by the function

$$A_m^2 = PS/4\pi r^2(t).$$

Since an assumption was made about the fact that the object moves according to a rectilinear trajectory, then for measurements during an artificial earth satellite's passage through the visibility range, the definition of three coordinates and three components of velocity is shown to be impossible. Actually, the position's planes, whose effect, as usual, is revealed in a generalized manner, intersect in this case not at the point, but along the circumference of the plane located in the traverse. The result of this is that the matrix determinants of the ACF second derivatives equal zero. According to the distance measurement results in one pass, it is possible to define only two coordinates and two components of the space vehicle's velocity for a rectilinear approximation of the trajectory. In view of this, it is advisable to present the SV's principle of movement in the following form, taking into account the assumptions adopted:

$$\xi_t = \xi + v_\xi t, \quad \eta_t = \eta + v_\eta t,$$
$$r = \sqrt{\xi_t^2 + \eta_t^2} = \sqrt{(\xi + v_\xi t)^2 + (\eta + v_\eta t)^2},$$

where ξ, η are the SV coordinates in the moment of time $t=0$.

The instantaneous range between the SV and the observer can also be represented in the form of a function

$$r = \sqrt{v_r^2 t^2 + 2 v_r r_0 t + r_0^2},$$

where

$$r_0^2 = \xi^2 + \eta^2; \quad v^2 = v_\xi^2 + v_\eta^2; \quad r_0 v_r = \xi v_\xi + \eta v_\eta,$$

ν_r is the radial velocity component at the moment of beginning the time count.

Greater simplicity and clearer representation are attained if the measurements are begun at the moment of traverse flight in the case where the distance between the SV and the observer reaches the minimum. At this moment, $\nu_r = 0$, and the instantaneous range is expressed by the formula $r = \sqrt{\rho^2 + \nu^2 t^2}$, where ρ is the traverse distance.

The partial derivatives according to defined coordinates ξ, η and components of velocity ν_ξ, ν_η are expressed by the following formulas: /115

$$\left.\begin{aligned} \frac{\partial r}{\partial \xi} &= \frac{\xi_t}{r}, & \frac{\partial r}{\partial \eta} &= \frac{\eta_t}{r}, \\ \frac{\partial r}{\partial v_\xi} &= \frac{\xi_t t}{r}, & \frac{\partial r}{\partial v_\eta} &= \frac{\eta_t t}{r}. \end{aligned}\right\} \tag{5.4.1}$$

The elements of the ACF second derivative matrix according to definable parameters are computed in the following manner:

$$Z''_{\xi\xi}(0) = -\frac{m^2 \Omega^2 PS}{4\pi v^2_{gr}} \int_0^T \frac{(\xi + v_\xi t)^2}{r^4} dt. \tag{5.4.2}$$

Deriving the corresponding calculations and inserting the notations

$$\varkappa = -m^2 \Omega^2 PS/4\pi\tau^2 gr, \quad x = vT/\rho,$$

for the case where measurements are begun in the traverse, we will obtain the formula

$$Z_{\xi\xi}^{\cdot}(0) = \frac{\varkappa}{2\rho v}\left[\frac{v_\xi^2}{v^2}\left(\operatorname{arctg} x - \frac{x}{1+x^2}\right) + \right.$$
$$\left. + \frac{\xi^2}{\rho^2}\left(\operatorname{arctg} x + \frac{x}{1+x^2}\right) + 2\frac{\xi v_\xi}{\rho v}\,\frac{x^2}{1+x^2}\right]. \qquad (5.4.3)$$

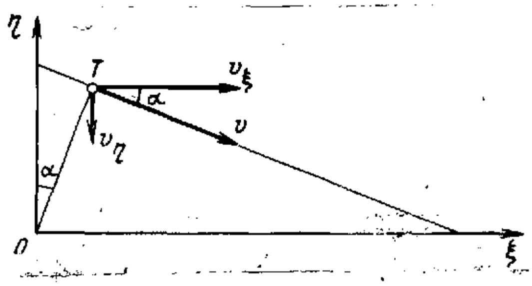

Fig. 5.1. The velocity vector and its components in the topocentric coordinate system used.

Turning to Fig. 5.1, in which the geometric relations are represented which take place at the moment of traverse flight, and taking into consideration that

$$v_\xi/v = \cos\alpha, \quad \xi/\rho = \sin\alpha,$$
$$v_\eta/v = \sin\alpha, \quad \eta/\rho = \cos\alpha, \qquad (5.4.4)$$

where α is the angle between the direction to the traverse point and axis η, and also defining /116

$$\operatorname{arctg} x - x/(1+x^2) = f_1(x), \quad \operatorname{arctg} x + x/(1+x^2) = f_2(x), \qquad (5.4.5)$$

we obtain the possibility of writing the computed second derivative in the following form:

$$Z''_{\xi\xi}(0)=\frac{\varkappa}{2\rho v}\left[f_1(x)\cos^2\alpha+f_2(x)\sin^2\alpha+\frac{x^2}{1+x^2}\sin 2\alpha\right]. \quad (5.4.6)$$

Taking into consideration that the formulas for the ACF second derivatives according to coordinates ξ and η are analogous in structure, it is possible to write

$$Z''_{\eta\eta}(0)=\frac{\varkappa}{2\rho v}\left[f_1(x)\sin^2\alpha+f_2(x)\cos^2\alpha+\frac{x^2}{1+x^2}\sin 2\alpha\right]. \quad (5.4.7)$$

Passing to the calculation of the ACF second derivatives with respect to the velocity components, we obtain the following relations:

$$Z''_{v_\xi v_\xi}(0)=\varkappa\int\limits_0^T\frac{t^2\xi_t^2}{r^4}dt=\frac{\varkappa\rho}{2v^3}\left\{f_1(x)\sin^2\alpha+\right.$$
$$\left.+\left[\log(1+x^2)-\frac{x}{1+x^2}\right]\sin 2\alpha+f_3(x)\cos^2\alpha\right\}, \quad (5.4.8)$$

where

$$f_3(x)=x+\frac{1}{2}\frac{x^2}{1+x^2}-\frac{3}{2}\operatorname{arctg} x. \quad (5.4.9)$$

By analogy, we have

$$Z''_{v_\eta v_\eta}(0)=\frac{\varkappa\rho}{2v^3}\left\{f_1(x)\cos^2\alpha+\left[\log(1+x^2)-\frac{x^2}{1+x^2}\right]\sin 2\alpha+\right.$$
$$\left.+f_3(x)\sin^2\alpha\right\}, \quad (5.4.10)$$

$$Z''_{v_\xi v_\eta}(0) = Z''_{v_\eta v_\xi}(0) = \\ = \frac{\varkappa\rho}{2v^3}\left[(x - \operatorname{arctg} x)\sin 2\alpha + \log(1+x^2) - \frac{x^2}{1+x^2}\right], \tag{5.4.11}$$

$$Z''_{\xi v_\xi}(0) = \frac{\varkappa}{2v^2}\left\{\left[\log(1+x^2) - \frac{x^2}{1+x^2}\right]\cos^2\alpha + \right. \\ \left. + f_1(x)\sin 2\alpha + \frac{x^2}{1+x^2}\sin^2\alpha\right\}, \tag{5.4.12}$$ /117

$$Z''_{\eta v_\eta}(0) = \frac{\varkappa}{2v^2}\left\{\left[\log(1+x^2) - \frac{x^2}{1+x^2}\right]\sin^2\alpha + \right. \\ \left. + f_1(x)\sin 2\alpha + \frac{x^2}{1+x^2}\cos^2\alpha\right\}, \tag{5.4.13}$$

$$Z''_{\xi v_\eta}(0) = Z''_{\eta v_\xi}(0) = \frac{\varkappa}{2v^2}\left\{[\log(1+x^2)]\frac{1}{2}\sin 2\alpha + f_1(x)\right\}, \tag{5.4.14}$$

$$Z''_{\xi\eta}(0) = Z''_{\eta\xi}(0) = \frac{\varkappa}{2\rho v}\left[(\operatorname{arctg} x)\sin 2\alpha + \frac{x^2}{1+x^2}\right]. \tag{5.4.15}$$

Thus, all elements of the matrix of ACF signal second derivative maximum values with respect to the components of coordinates and velocity have been computed.

Since the relation takes place

$$\int_{-T}^{T} f(t)\,dt = \int_{0}^{T} f(t)\,dt - \int_{0}^{-T} f(t)\,dt, \tag{5.4.16}$$

then for the symmetry of the measured segment of the trajectory, the matrix of ACF second derivatives is reduced to the form of (5.4.17). It is interesting that with $\alpha=0$, $\alpha=45°$ and $\alpha=90°$, respectively, the first and fourth, first and second, second and third columns of matrix (5.4.17) become proportional to each other. This attests to the fact that in the cases mentioned, the transformation of the given matrix, meaning the definition of both coordinates and both components of velocity, becomes impossible.

In the cases cited, only three of the four definable values are successfully defined. Let us examine, in particular, the case where $\alpha=0$ and calculate the correlation matrix of errors in defining the coordinates and vector velocity modulus for this case.

The initial matrix of second derivatives in the given case is in the form of (5.4.18):

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$$Z''(0)=\varkappa\begin{Vmatrix} \frac{1}{\rho v}[f_1(x)\cos^2\alpha+f_2(x)\sin^2\alpha] & \frac{1}{\rho v}\operatorname{arctg}x\sin 2\alpha & \frac{f_1(x)}{v^2}\sin 2\alpha & \frac{1}{v^2}f_1(x) \\ \frac{1}{\rho v}\operatorname{arctg}x\sin 2\alpha & \frac{1}{\rho v}[f_1(x)\sin^2\alpha+f_2(x)\cos^2\alpha] & \frac{1}{v^2}f_1(x) & \frac{f_1(x)}{v^2}\sin 2\alpha \\ \frac{f_1(x)}{v^2}\sin 2\alpha & \frac{f_1(x)}{v^2} & \frac{\rho}{v^3}[f_1(x)\sin^2\alpha+f_3(x)\cos^2\alpha] & \frac{\rho}{v^3}\sin 2\alpha\times(x-\operatorname{arctg}x) \\ \frac{f_1(x)}{v^2} & \frac{f_1(x)}{v^2}\sin 2\alpha & \frac{\rho}{v^3}(x-\operatorname{arctg}x)\times\sin 2\alpha & \frac{\rho}{v^3}[f_1(x)\cos^2\alpha+f_3(x)\sin^2\alpha] \end{Vmatrix} \quad (5.4.17)$$

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$$Z''(0)=\varkappa\begin{Vmatrix} \frac{1}{\rho v}f_1(x) & 0 & 0 \\ 0 & \frac{1}{\rho v}f_2(x) & \frac{1}{v^2}f_1(x) \\ 0 & \frac{1}{v^2}f_1(x) & \frac{\rho}{v^3}f_3(x) \end{Vmatrix}. \quad (5.4.18)$$

The error measurement correlation matrix is related to the ACF second derivative matrices by relation (3.1.16) which, in the absence of a priori data, results in the form

$$B = \| b_{ij} \| = -\frac{N_0}{2} [Z''(0)]^{-1}, \qquad (5.4.19)$$

where N_0 is the spectral noise density.

Thus, the elements of the second derivative correlation matrix are expressed by a formula in the form

$$b_{ij} = -\frac{N_0}{2} \frac{[Z''_{ij}(0)]^*}{\det Z''(0)},$$

where $[Z''_{ij}(0)]^*$ is the signed minor of the corresponding element; $\det Z''(0)$ is the determinant of the second derivative matrix which is equal to the following magnitude:

$$\det Z''(0) = z''_{11} (z''_{22} z''_{33} - z''^{2}_{23}). \qquad (5.4.20)$$

Using the formulas derived, we obtain the following expressions for the elements of the correlation matrix:

$$b_{11} = \frac{2 N_0 v^2_{gr}}{m^2 \Omega^2 PS} \rho v f_4(x), \qquad (5.4.21)$$

where

$$b_{32} = \frac{v^2_{gr} 2\pi N_0}{m^2 \Omega^2 PS} \rho v f_5(x), \qquad (5.4.22)$$

$$f_4(x) = [f_1(x)]^{-1}; \qquad (5.4.23)$$

where

$$f_5(x) = \left\{ f_2(x) \left[1 - \frac{f_1^2(x)}{f_2(x) f_3(x)} \right] \right\}^{-1}; \qquad (5.4.24)$$

$$b_{33} = \frac{2\pi N_0 v_{gr}^2 v^3}{m^2 \Omega^2 P S \rho} f_6(x), \tag{5.4.25}$$ /120

where

$$f_6(x) = \left\{ f_3(x) \left[1 - \frac{f_1^2(x)}{f_2(x) f_3(x)} \right] \right\}^{-1}; \tag{5.4.26}$$

$$b_{12} = 0, \; b_{13} = 0, \quad b_{23} = \frac{2\pi N_0 v_{gr}^2}{m^2 \Omega^2 P S} v^2 f_7(x), \tag{5.4.27}$$

where

$$f_7(x) = \{ f_1(x) [1 - f_2(x) f_3(x) / f_1^2(x)] \}^{-1}. \tag{5.4.28}$$

In this way, the correlation matrix of errors in defining the coordinates and velocity has the following form:

$$B = \frac{2\pi N_0 v_{gr}^2}{m^2 \Omega^2 P S} \begin{Vmatrix} \rho v f_4(x) & 0 & 0 \\ 0 & \rho v f_5(x) & v^2 f_7(x) \\ 0 & v^2 f_7(x) & (v^3/\rho) f_6(x) \end{Vmatrix}. \tag{5.4.29}$$

Table 5.1

x	0,1	0,25	0,50	0,75	1,0
f_1	$6{,}588 \cdot 10^{-4}$	$9{,}684 \cdot 10^{-3}$	$6{,}365 \cdot 10^{-2}$	0,1635	0.2854
f_2	0,1987	0,4803	0,8636	1,124	1,285
f_3	$3{,}944 \cdot 10^{-6}$	$3{,}581 \cdot 10^{-4}$	$9{,}087 \cdot 10^{-3}$	$4{,}950 \cdot 10^{-2}$	0,1438
f_4	1518	103,2	15,71	6,116	3,504
f_5	11,28	4,579	2,402	1,714	1,390
f_6	$5{,}684 \cdot 10^{5}$	6141	229,0	38,91	12.43
f_7	−1885	−123,8	−16,88	−5,662	−2,760

The values of functions $f_1(x) \div f_7(x)$ for a number of values of the argument are shown in Table 5.1, and the graphs of these functions are given in Figs. 5.2. and 5.3. In extreme cases where the length of the measured segment becomes sufficiently large, the correlation matrix of errors takes the form

$$\lim_{x\to\infty} B = \frac{2\pi N_0 v_{gr}^2}{m^2\Omega^2 PS} \begin{Vmatrix} 2\pi^{-1}\rho v & 0 & 0 \\ 0 & 2\pi^{-1}\rho v & -v^2(2x)^{-1} \\ 0 & -v^2(2x)^{-1} & \dfrac{v^3}{2\rho x} = \dfrac{v^2}{2T} \end{Vmatrix}. \quad (5.4.30)$$ /121

We will sum up the computations and form some basic conclusions.

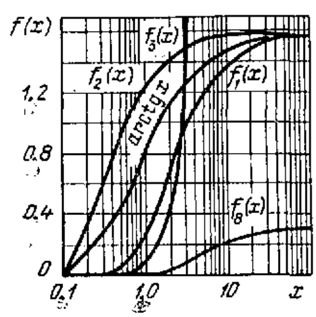

Fig. 5.2. Graph of functions $f_1(x) \div f_3(x)$, $f_8(x)$.

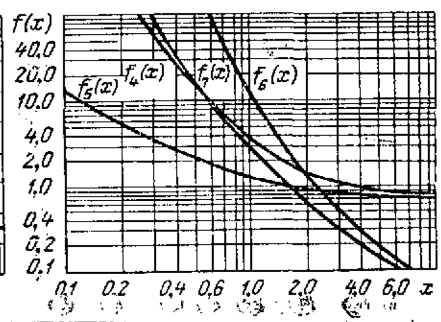

Fig. 5.3. Graph of functions $f_4(x) \div f_7(x)$.

2.0	3,0	5,0	10	∞
0,7071	0,9490	1,181	1,372	$2^{-1}\pi$
1,507	1,549	1,566	1,570	$2^{-1}\pi$
1,079	2,553	6,072	15,68	$2x-1,5\pi$
1,414	1,054	0,8467	0,7288	$2\pi^{-1}$
0,9583	0,8360	0,7485	0,6896	$2\pi^{-1}$
1,339	0,5072	0,1930	$6,903\cdot10^{-2}$	$(2x)^{-1}$
−0,6283	−0,3108	−0,1456	$6,032\cdot10^{-2}$	$-(2x)^{-1}$

First of all, it is necessary to note that the error dispersions in defining the topocentric coordinates and velocity are directly proportional to the spectral noise density and inversely proportional to the power of the signal transmitted by the on-board transmitter, to the square of percentage modulation and the square of the frequency to which the position's planes are assigned.

The errors in defining the coordinates are proportional, /122
moreover, to the velocity and traverse distance and are functions of the relation between the length of the measured trajectory segment and the traverse distance.

If the length of the measured segment is long, the errors in defining the SV's position in the trajectory exceed the errors in defining the traverse distance; however, where the length of the measured segment is sufficiently great, equalization of the component errors of measurement takes place, and the error ellipse turns into a circle.

Errors in defining the velocity are directly proportional to the square of the velocity modulus, inversely proportional to the duration of the measurements, and depend on the relationship between the length of the measured trajectory segment and the traverse distance.

The correlation matrices (5.4.29) and (5.4.30) describe the potential accuracy of the phase range method of defining the coordinates and velocity of the SV. We will now calculate the correlation matrix of errors with the Doppler method of measuring.

The elements of the ACF signal second derivative matrix with Doppler measurements are expressed by the formulas

$$Z''_{ij}(0)=-\frac{k^2}{2}\int\limits_{-T}^{T}\frac{\partial r}{\partial q_i}\frac{\partial r}{\partial q_j}A^2\,dt+$$
$$+\frac{k^2}{4E}\left(\int\limits_{-T}^{T}\frac{\partial r}{\partial q_i}A^2\,dt\right)\left(\int\limits_{-T}^{T}\frac{\partial r}{\partial q_j}A^2\,dt\right). \tag{5.4.31}$$

Using these, with symmetry of the measured segment, we derive the following relations:

$$Z''_{1\xi\xi}(0)=Z''_{\xi\xi}(0)=\frac{\varkappa_1}{\rho v}f_1(x),$$

$$Z''_{1\eta\eta}(0)=\frac{\varkappa_1}{\rho v}\left[f_2(x)-\frac{2x^2}{(1+x^2)\,\mathrm{arctg}\,x}\right]=\frac{\varkappa_1}{\rho v}f_8(x), \tag{5.4.32}$$

$$Z''_{1vv}(0)=\frac{\varkappa_1\rho}{v^3}\left[f_3(x)-2\frac{\left(\mathrm{arcsh}\,x-\frac{x}{\sqrt{1+x^2}}\right)^2}{\mathrm{arctg}\,x}\right]=\frac{\varkappa_1\rho}{v^3}f_9(x), \tag{5.4.33}$$

$$Z''_{1\xi\eta}(0)=0,\quad Z''_{1\xi v}(0)=0,$$

$$Z''_{1\eta v}(0)=\frac{\varkappa_1}{v^2}\left[f_1(x)-\frac{2x}{\sqrt{1+x^2}\,\mathrm{arctg}\,x}\left(\mathrm{arcsh}\,x-\frac{x}{\sqrt{1+x^2}}\right)\right]=\frac{\varkappa_1}{v^2}f_{10}(x), \tag{5.4.34}$$

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where $\varkappa_1 = -k^2PS/8\pi$.

Elements of the error correlation matrix are represented by the following formulas:

$$b_{111}=\frac{4\pi N_0}{k^2PS}\rho v f_4(x); \tag{5.4.35}$$

$$b_{122}=\frac{4\pi N_0}{k^2PS}\rho v f_{11}(x), \tag{5.4.36}$$

where

$$f_{11}(x)=\left\{f_8(x)\left[1-\frac{f_{10}^2(x)}{f_8(x)f_9(x)}\right]\right\}^{-1}; \tag{5.4.37}$$

$$b_{133}=\frac{4\pi N_0}{k^2PS}\frac{v^3}{\rho}f_{12}(x), \tag{5.4.38}$$

where

$$f_{12}(x)=\left\{f_9(x)\left[1-\frac{f_{10}^2(x)}{f_8(x)f_9(x)}\right]\right\}^{-1}; \tag{5.4.39}$$

$$b_{123} = -\frac{4\pi N_0}{k^2 PS} v^2 f_{13}(x), \tag{5.4.40}$$

where

$$f_{13}(x) = \left\{ f_{10}(x) \left[1 - \frac{f_8(x) f_9(x)}{f_{10}^2(x)} \right] \right\}^{-1}. \tag{5.4.41}$$

Thus, the error correlation matrix obtains the following form:

$$B_1 = \frac{4\pi N_0}{k^2 PS} \begin{Vmatrix} \rho v f_4(x) & 0 & 0 \\ 0 & \rho v f_{11}(x) & v^2 f_{13}(x) \\ 0 & v^2 f_{13}(x) & v^3 \rho^{-1} f_{12}(x) \end{Vmatrix}. \tag{5.4.42}$$

The values of functions $f_8(x) \div f_{13}(x)$ are given in Table 5.2, and the graphs of functions $f_4(x)$, $f_{11}(x) \div f_{13}(x)$ are represented in Fig. 5.4.

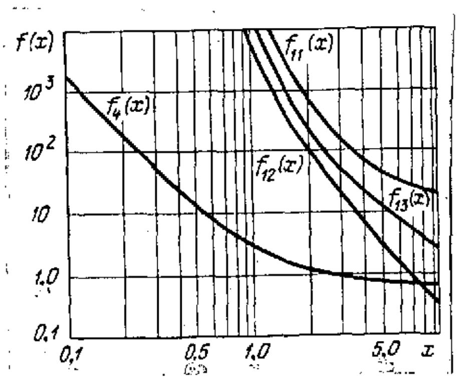

Fig. 5.4. Graph of functions $f_4(x)$, $f_{11}(x) \div f_{13}(x)$.

The chief characteristic of the Doppler method error correlation matrix (5.4.42) lies in the fact that the dependence of the error dispersion in defining the SV's position in the trajectory on the length of the measured segment is identical with the corresponding dependence of the range-measuring method, although the dispersions themselves, other conditions being equal, differ from each other by $m^2\Omega^2/\omega^2$ times. As for errors in defining the traverse distance and velocity, they differ from errors in defining these values by the range-measuring method not only by size, but also by the nature of the dependence on the length of the measured segment. Errors in defining the traverse distance by /124 /125

Table 5.2

x	0,1	0,25	0,50	0,75	1,0	2,0	3,0	5,0	10	∞
f_8	$4{,}365\cdot 10^{-7}$	$3{,}888\cdot 10^{-5}$	$9{,}234\cdot 10^{-4}$	$4{,}622\cdot 10^{-3}$	$1216\cdot 10^{-2}$	$6{,}200\cdot 10^{-2}$	0,1079	0,1655	0,2241	$(2\pi)^{-1}(\pi^2-8)$
f_9	$1{,}754\cdot 10^{-6}$	$1{,}596\cdot 10^{-4}$	$4{,}071\cdot 10^{-3}$	$2{,}253\cdot 10^{-2}$	$6{,}647\cdot 10^{-2}$	0,5337	1,342	3,489	10,23	$2x-\frac{3\pi}{2}-\frac{4}{\pi}(\operatorname{arsh} x)^2$
f_{10}	$-8{,}749\cdot 10^{-7}$	$-7{,}878\cdot 10^{-5}$	$-1{,}939\cdot 10^{-3}$	$-1{,}020\cdot 10^{-2}$	$-2{,}839\cdot 10^{-2}$	−0,1802	−0,3722	−0,7207	−1,338	$\frac{\pi}{2}-\frac{4}{\pi}\operatorname{arsh} x$
f_4	1518	103,2	15,71	6,116	3,504	1,414	1,054	0,8467	0,7288	$2\pi^{-1}$
f_{11}	$4{,}872\cdot 10^{10}$	$1{,}638\cdot 10^{9}$	$4{,}852\cdot 10^{6}$	$2{,}220\cdot 10^{5}$	$3{,}197\cdot 10^{4}$	882,6	212,8	60,29	20,33	$2\pi(\pi^2-8)^{-1}$
f_{12}	$1{,}213\cdot 10^{10}$	$3{,}989\cdot 10^{8}$	$1{,}101\cdot 10^{6}$	$4{,}554\cdot 10^{4}$	5847	102,5	17,12	2,859	0,4454	$(2x)^{-1}$
f_{13}	$2{,}431\cdot 10^{10}$	$8{,}083\cdot 10^{8}$	$2{,}311\cdot 10^{6}$	$1{,}005\cdot 10^{5}$	$1{,}365\cdot 10^{4}$	298,1	59,03	12,45	2,659	$4(\pi^2-8)^{-1}\frac{\operatorname{arsh} x}{x}$
f_{14}	$6{,}571\cdot 10^{4}$	$1{,}891\cdot 10^{4}$	1421	369,9	151,6	30,35	15,95	8,974	5,430	$\pi(\pi^2-8)^{-\frac{1}{2}}$
f_{15}	146,1	254,9	69,32	34,21	21,69	8,750	5,810	3,849	2,540	1
f_{16}	3591	2555	370,0	133,2	70,33	21,78	13,78	9,249	6,639	$2\sqrt{\frac{2\operatorname{arsh} x}{\pi^2-8}}$

the range-measuring method are always less than the errors in defining the SV's position in the trajectory, and their equality approaches the limit only with $x \to \infty$. In the case of the Doppler method, errors in defining the traverse distance are always greater than errors in defining the SV's position in the trajectory. The relations of the mean square errors in defining the coordinates and velocity by Doppler and range-measuring methods are correspondingly equal to

$$(m\Omega/\omega)f_{14},\quad (m\Omega/\omega)f_{15},\quad (m\Omega/\omega)f_{16}.$$

The functions $f_{14}=\sqrt{f_{11}f_5^{-1}},\ f_{15}=\sqrt{f_{12}f_6^{-1}},\ f_{16}=\sqrt{f_{13}f_7^{-1}}$ are shown in Table 5.2. If the length of the measured trajectory segment approaches infinity, then the limit of error correlation matrix (5.4.42) is the matrix

$$\lim_{x\to\infty} B_1=\frac{4\pi N_0}{k^2 PS}\left\|\begin{matrix} \frac{2}{\pi}\rho v & 0 & 0 \\ 0 & \frac{2\pi}{\pi^2-8}\rho v & \frac{4\,\mathrm{arsh}\,x}{(\pi^2-8)x}v^2 \\ 0 & \frac{4\,\mathrm{arsh}\,x}{(\pi^2-8)x}v^2 & \frac{v^3}{2\rho x}=\frac{v^2}{2T} \end{matrix}\right\|, \qquad (5.4.43)$$ /126

which, in the nature of the functions, is not very different from matrix (5.4.30). Matrices (5.4.43) and (5.4.30) basically differ in the error dispersions in defining the traverse distance.

5.5. Concerning the Informativeness of Different Trajectory Segments

In analyzing the results of the research presented in the previous section, it is possible to express some considerations about the informativeness of the different sections of the measured trajectory segments.

It is obvious that the specific increase in information about the parameters of motion which appears in different sections of the measured trajectory segment of the SV must differ. Energy conditions at the moment of measurement, the velocity of the angular change of the gradient to the planes of the position, and the position of the observer with respect to the measured segment exert an influence on the magnitude of the increase, evidently. In view of the complexity of the phenomena, we must make a special examination of this question.

For obtaining a sufficiently clear picture of the phenomena in the analysis, we must eliminate the influence of the coordinate transformations and, namely, according to this principle, the coordinates and components of velocity in a rectangular system of coordinates are chosen as the definable parameters.

The first question which must be answered before we begin to examine the problem of evaluating the informativeness of the measurements is that which concerns which value should be accepted as the informativeness measure of one or another section of trajectory. At first glance, it seems that the most reasonable measure is a value which shows how much the errors in defining some parameter of motion decrease in measuring a segment of unit length. However, such a measure proves to be practically unsuitable, for two reasons. First, it is unsuitable in that it does not satisfy the condition of additivity, which this type of measure must obviously satisfy.

Let us explain what this implies. It would be desirable if the informativeness measure were increasing functions of the length of the section and if the measure of informativeness of two sections were equal to the sum of the informativeness measures of each of them.

If a value proportional to a decrease in the measurement error dispersion in a section of unit length is chosen as the informativeness measure, then such a measure will not satisfy the condition of additivity. /127

Another shortcoming of the measure examined consists of the fact that it involves the necessity of choosing an infinitely large value as the dispersion origin. Actually, with respect to the degree of shortening the measured segment, the errors increase, approaching infinity. The choice of such an origin is connected with great drawbacks: the dispersions diminish to values large in magnitude, which attests to the efficient functioning of the system; at the same time, intolerably large absolute values of the measurement errors correspond, because in subtracting a large but finite value from the infinitely large value, we will obtain, as usual, a value which describes the increase in the measurement accuracy on a section of trajectory of unit length.

Thus, evaluating the informativeness of a trajectory segment by decreasing the error dispersion on it is shown to be impractical. It is evident that a value which describes an increase in measurement accuracy on a trajectory segment of unit length can serve as a more advantageous measure of informativeness.

The increase in accuracy can be estimated by the increase in the value of inverse measurement error dispersion to a unit segment, i.e., by an increase of the value of matrix elements

$$B_{ii}^{-1} = -\frac{2}{N_0}\sum_k \frac{E_k}{N_0 + E_k} Z''_{ii}(0).$$

In the case of signals whose correlation interval of initial phase fluctuations are sufficiently high, it is possible to take the increase of the value $-Z''_{ii}(0)N_0^{-1}$ as a convenient measure, or simply the value

$$-\frac{\partial}{\partial x} Z''_{ii}(0),$$

which we will designate by the letters I_i. These measures satisfy the condition of additivity, and the origin of each of them coincides with zero. Thus, it is practical to examine a derivative according to the length of the measured trajectory segment from the maximum value of the ACF second derivative with respect to a definable parameter as the informativeness measure of the given trajectory section.

Using the measure introduced, we will evaluate the informativeness of different segments of the SV's trajectory according to their relation to different parameters of motion.

Determining the SV's Position in the Trajectory

The potential accuracy of defining coordinate χ, which describes the position of the SV in the trajectory, is evaluated by the maximum value of the ACF second derivative with respect to this coordinate which is identical in form both for range-measuring and Doppler methods.

The informativeness of different trajectory segments in defining this coordinate is described by the magnitude

$$I_x = -\frac{\partial}{\partial x} Z''_{xx}(0) = \chi_1 f_1'(x) \simeq \chi_1 \frac{2x^2}{(1+x^2)^2}, \tag{5.5.1}$$

where $\chi_1 = x/\rho\nu$.

Table 5.3 /128

x	0	0,5	1,0	2,0	3,0	5,0	10,0
$f_1'(x)$	0	0.32	0,50	0,32	0.18	0,07	0,02
$f_2'(x)$	2.00	1,27	0,50	0,08	0,02	0,003	0.00
$f_3'(x)$	0	0,04	0,25	0,64	0.81	0,92	1,00
$f_8'(x)$	0	$3,6\cdot 10^{-3}$	0,18	0,257	0,02	0,01	$3,4\cdot 10^{-3}$
$(1+x^2)^{-3}$	1	0.51	0,12	0,008	10^{-3}	—	—
$x^2(1+x^2)^{-3}$	0	0.13	0.13	0,032	0,009	0,004	10^{-4}

Note that χ is a dimensionless generalized coordinate, equal to $\nu T/\rho$. If the need arises for evaluating the informativeness of a trajectory segment measured by normal linear units ξ, then it is necessary to take into account that a relation takes place between the corresponding measures of informativeness

$$I_\xi = \frac{1}{\rho} I_x.$$

The numerical values of function $f_1'(x)$ are shown in Table 5.3, and its graph is given in Fig. 5.5.

From the table and graph, we can see that, from the point of view of defining the SV's position in the trajectory,

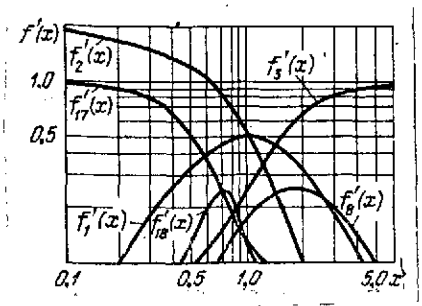

Fig. 5.5. Informativeness of different trajectory segments with relation to the different components of coordinates and velocity.

the segment removed from the traverse in the length of the traverse distance is the most informative. Informativeness which exceeds half of the maximum is attained within the limits of the interval of values x from $\sqrt{3-2\sqrt{2}}$ $\sqrt{3+2\sqrt{2}}$.

According to measurements near the traverse and at distances exceeding the triple value of the traverse distance, the SV's position in the trajectory is poorly determined.

In using the data cited, we should /129
keep in mind the fact that they are completely related to the range-measuring method. As for the Doppler method, they are applicable to it only if they fulfill the conditions for executing this method, which amounts to the fact that the measured trajectory segment must include two segments identical in length located in the trajectory symmetrical to the traverse point.

Determination of the Traverse Distance by the Range-Measuring Method

We can judge the informativeness of different trajectory segments in a given case by the magnitude $I_\rho = -(d/dx)Z''_{\rho\rho}(0)$ which in the case considered is equal to $I_\rho = \chi_1 f_2(\chi) = 2\chi_1/(1+x^2)^2$.

The values of function $f_2'(x)$ are shown in Table 5.3 and Fig. 5.5.

The trajectory segment near the traverse is the most informative from the point of view of defining the traverse distance; the trajectory segment whose length is somewhat greater than the value of the traverse distance has informativeness which exceeds half of its value on the traverse.

Determining the Traverse Distance by the Doppler Method

The informativeness of different trajectory segments is characterized in this case by the function

$$I_\rho = \chi_2 f_8(x) = \chi_2 (x - \operatorname{arctg} x)^2/(1+x^2)^2 (\operatorname{arctg} x)^2,$$

where $\chi_2 = x_1/\rho\nu$. The values of this function are shown in Table 5.3 and Fig. 5.5.

We can see from the graph and table that the trajectory segment removed from the traverse at double the value of the traverse distance is the most informative. The Doppler measurements in the circumtraverse segment of trajectory are only slightly effective.

Determining the Components of the Velocity Vector

Data given in the previous section allow judging the informativeness of the velocity measuring process by using signals with known and unknown initial phases. In this connection, for a signal with a known initial phase, the informativeness of defining the longitudinal and transverse components is successfully evaluated.

The informativeness of the trajectory for defining the transverse component is evaluated by the same derivative of the function $f_1(x)$, which describes the conditions for defining the SV's position in the trajectory (see Table 5.3 and Fig. 5.5). Trajectory segments distant from the traverse point on the length of the traverse distance are more favorable for defining the transverse components of velocity.

In order to obtain an idea about measurement conditions more favorable from the point of view of defining the longitudinal components of velocity, it is necessary to compute the derivative /130
of function $f_3(x)$ according to x. This derivative is equal to

$$f_3'(x) = x^4(1+x^2)^{-2}.$$

The derivative's values are given in Table 5.3 and in Fig. 5.5.

The trajectory segments farthest from the traverse are distinguished by greater informativeness. Measurements on trajectory segments far from the traverse point at distances greater than the traverse have a practical meaning.

As we see from formula (5.4.18), the derivative of $f_3'(x)$ also describes the conditions which are more favorable for defining the velocity vector modulus.

We can judge the informativeness of different trajectory segments in defining the velocity modulus by means of a signal with known initial phase by the value of the derivative $f_9'(x)$. The analytical expression for this derivative is rather tedious and difficult to analyze; however, from comparing functions $f_3(x)$ and $f_9(x)$, whose values are given in Tables 5.1 and 5.2, we can see that when x increases, both functions increase monotonically. With a small x, the values of the second function are approximately two times less than the corresponding values of the first. A numerical evaluation of the values of the derivative of $f_9(x)$ shows that it differs little from $f_3'(x)$, both with respect to the nature of the function and the numerical values. Therefore, it is possible to consider that in a first approximation of the informativeness, the properties of different trajectory segments for defining the SV's velocity for a signal with an unknown phase are approximately the same as for a signal with a known initial phase.

Defining the Parameters of Motion by Azimuth-Scale Methods

For completing the picture, it is also expedient to evaluate the informativeness of different segments of trajectory by azimuth-scale methods. Since the potential accuracy of azimuth-scale methods on one pass were not previously evaluated, we will cite the formulas for the ACF second derivatives of a signal received from a SV moving evenly along a rectilinear trajectory. Combining the origin of time with the moment of passing the traverse and using the notation accepted in §5.4 for angle γ, equal to the angular distance between the traverse point and the point of the SV's position in the trajectory, we will obtain

$$\gamma = \operatorname{arctg} \frac{vt}{\rho} = \operatorname{arctg} \zeta.$$

The angle's derivatives with respect to the definable parameters of motion will be equal to

$$\frac{\partial \gamma}{\partial \zeta} = \frac{1}{1+\zeta^2}, \quad \frac{\partial \gamma}{\partial \rho} = -\frac{1}{\rho}\,\frac{\zeta}{1+\zeta^2}, \quad \frac{\partial \gamma}{\partial v} = \frac{1}{v}\,\frac{\zeta}{1+\zeta^2}.$$

Assuming that the definition of the angular coordinates is produced by means of a parabolic antenna with a circular aperture having diameter D_0, we obtain the following expressions for the maximum values of the ACF second derivatives from formula (4.4.6): /131

$$Z''_{xx}(0) = \frac{\varkappa_2}{\rho^3 v} \int_0^x \frac{d\zeta}{(1+\zeta^2)^3},$$

$$\varkappa_2 = \frac{k^2 D_0^4 P}{1024}, \quad \zeta = \frac{vt}{\rho}, \; x = \frac{vT}{\rho}.$$

$$Z''_{\rho\rho}(0) = \frac{\varkappa_2}{\rho^3 v} \int_0^x \frac{\zeta^2 d\zeta}{(1+\zeta^2)^3},$$

$$Z''_{vv}(0) = \frac{\varkappa_2}{\rho v^3} \int_0^x \frac{\zeta^2 d\zeta}{(1+\zeta^2)^3}.$$

The informativeness of the measured trajectory segment is described by derivatives of the maximum values of ACF second derivatives with respect to generalized coordinate x, which in the given case are equal to

$$I_x = \frac{\varkappa_2}{v\rho^3} \frac{1}{(1+x^2)^3} = \frac{\varkappa_2}{v\rho^3} f'_{17}(x),$$

$$I_\rho = \frac{\varkappa_2}{v\rho^3} \frac{x_2}{(1+x^2)^3} = \frac{\varkappa_2}{v\rho^3} f'_{18}(x),$$

$$I_v = \frac{\varkappa_2}{\rho v^3} \frac{x_2}{(1+x^2)^3} = \frac{\varkappa_2}{\rho v^3} f'_{18}(x).$$

The values of these functions are given in Table 5.3 and their graph is shown in Fig. 5.5. In examining the table and graph, we can form the following conclusions. In defining the SV's position in the trajectory by azimuth-scale methods, the close traverse segment has the most informativeness. Its length is comparatively small: the informativeness exceeds half of its maximum value within the limits of a segment whose length is equal to the traverse distance, which is somewhat less than the length of the most informative part of the trajectory with range measurements.

The traverse distance and the velocity modulus are more effectively determined at a distance of 0.75 ρ from the traverse.

As we expected, the informativeness of azimuth-scale measurements decreases much more rapidly with an increase of the traverse distance than the informativeness of range measurements does.

In this respect, azimuth-scale methods are even inferior to Doppler methods. Naturally, the accuracy of azimuth-scale methods as a function of the distance to the SV appears weaker if the linear dimensions or base of the antenna system is rather large. In comparing azimuth-scale with range-measuring methods, it is necessary to take into consideration that the accuracy of these systems is usually limited not by the fluctuational errors, but by other errors, in particular by refraction errors. An important quality of azimuth-scale methods lies in the fact that it is possible to define not only the SV's position in a plane which passes through the trajectory and observation point by means of these methods, but also to define the SV's shift with respect to this plane. Neither range-measuring nor Doppler methods permit doing this. /132

In concluding the examination of the informativeness of different sections of a measured trajectory segment, we should note several features of the results obtained. First of all, we can see that it is impossible to speak of the informativeness of a trajectory segment in general, without referring to the definable parameter. From the materials cited, we can see that the different trajectory segments furnish data which differ substantially with respect to the parameters of motion.

The near-traverse measurements are useful from the point of view of defining the traverse distance by range-measuring methods and the SV's shift along the trajectory and across it by azimuth-scale methods.

The SV's position in the trajectory is defined by the range-measuring method and the traverse distance is determined by the azimuth-scale method; definition of all parameters of motion by the Doppler method is more expediently done by removing the traverse point to an order of magnitude of the traverse distance.

Thus, despite the fact that measurements near the traverse are suitable for the energy relation, they are unsuitable in a number of cases from the point of view of attaining high accuracy in measuring the parameters of motion.

From the examination presented, it follows that for attaining higher accuracy in defining a greater number of parameters of motion with short-duration measurements by range-measuring and Doppler methods, the entire cycle of measurements is practically divided into 2-3 times (or 2-3 separate measurements) so that one of the times would coincide with the period of the SV's stay in the traverse region, and the others would correspond to the rather large distance of the SV from the traverse.

The data cited allow us to judge the informativeness of different trajectory segments in measuring during one pass in the visibility range, assuming a linear approximation of the measured trajectory segment. Questions arise about what will result if the measurements are carried out not during one, but during several passes, and what the negative consequences of the assumption about the trajectory's linearity will be.

The answer to the first question is clear. For measurements during one pass, two coordinates of the SV and two components of its velocity in the plane which includes the trajectory and the observer are defined. Measurements during another pass allow defining the same four values in another plane which, generally speaking, is not coplanar with the first.

In processing the results of measurements during the two passes and computing a priori data about the orbit, it is also possible to select values for the initial conditions with respect to any moment of time which better correspond to a more accurate orbit and with which the SV, at the moment of passing the traverses, will pass through the point found in the process of measuring during separate passes, and will have a velocity at these points whose components will coincide with the more accurate values of the corresponding velocity components.

In this way, it is clear that the accuracy in defining the parameters of motion on several passes will be higher, other conditions being equal, with a more precise definition of the corrections to the coordinates and velocity components on each of the passes separately. The materials cited give the answer with respect to more favorable conditions for defining the different components of the parameters of motion during separate passes. The question concerning the choice of the most favorable observation conditions during subsequent passes is a separate problem which is outside the scope of the present investigation. It can be expected that the final results will be more precise if the angle of intersection of the plane where the components of the coordinates and velocity, made more accurate during the separate passes, are ordered, is closer to a right angle. /133

The question about how much the data presented here differs from the informativeness of an elliptical or circular trajectory, with respect to the informativeness of the different sections of the measured linear trajectory segment, also requires special examination.

Actual circular and elliptical trajectories will, naturally, differ from the rectilinear trajectories which were discussed with respect to their informational properties, although a linear approximation, evidently, allows us to judge rather reliably the qualitative picture of phenomena which result from defining circular and elliptical trajectories with low flight altitudes and small eccentricities.

The results mainly give a qualitative idea of the informativeness of different sections of the measured trajectory segment and can serve as a starting point for a more detailed examination of this question, for example, by methods of numerical analysis.

In analyzing the results of an informativeness evaluation, it is again necessary to consider the question of the choice of the informativeness measures. At first glance, it seems that the informativeness measure used does not always completely and reliably reflect the informativeness of a trajectory segment, since the measure examined is not strictly connected with the value of the decrease in the error dispersion. Difficulties with inverting the ACF second derivative matrix can arise with its use. In particular, in making, for example, range measurements on a close traverse segment of trajectory which is more informative for defining the traverse distance, we are not in a position to obtain any information about the two-dimensional or three-dimensional vector of the parameters of motion due to the fact that the matrix of ACF second derivatives, according to the initial conditions in the given case, is not yielded by inversion. Therefore, it seems that the most accurate representation of informativeness and the actual picture of the measurement results are given only by the value of the decrease in error dispersions in a measurement segment of unit length. However, it is obvious that similar doubts do not have serious foundations, and in reality the informativeness measure, equal to an increase in the measurement accuracy of a given parameter in a measured segment of unit length, offer an objective and true concept of the effectiveness of the measuring process.

The difficulties which arise in inverting the ACF second derivative matrix are reasonable and explicable. What is more, the indicators of informativeness examined clearly show in which conditions the matrix of second derivatives will yield an inversion, in which it will not, and which measures must be carried out for obtaining its invertibility. Actually, returning to the example cited, we should note that, as the measure of informativeness shows, the close traverse segment in range measurements conveys information only about one geometric value -- the traverse distance, and every experiment for evaluating the errors in defining two or three coordinates are doomed to failure. Therefore, inverting a three-dimensional matrix of ACF second derivatives is impossible.

On the other hand, as the informativeness indicators indicate, it is possible to obtain information about two geometric /134 values -- the traverse distance and the SV's position in the trajectory -- by the given range measurements on the traverse and at a defined distance from the traverse. Therefore, a two-dimensional matrix of ACF second derivatives of a signal received during two spaced time intervals (near the traverse and removed from it to the value of the traverse distance) yield an inversion.

From the considerations cited, it becomes clear that the informativeness measure used in a given operation offers the possibility of a farily detailed, reliable, and objective evaluation of a quantity of data which can be obtained in the process of measuring a given trajectory segment, and this measure can be recommended for actual use.

Chapter 6

ANALYZING THE PROPERTIES FOR DEFINING DIFFERENT SYSTEMS OF PARAMETERS OF MOTION

6.1. Introduction

Among the many systems of parameters which uniquely describe the SV's position in a phase range [26], we can distinguish the following:

-- components of the coordinates and the velocity vector (the initial conditions of motion) of the SV in some geocentric or topocentric systems of coordinates with respect to a defined moment of time t_0;

-- Keplerian and similar orbital elements;

-- canonical parameters.

The choice of a concrete system of parameters is dictated both by the content of navigational (geodetic) determinations and their method and by the geometric properties of space. In particular cases, the properties of space are decisive.

In satellite navigation and geodesy using a fixed [illegible] system of reference, values which define the spatial position of the observer in the coordinate system selected appear as the evaluated parameters.

Together with the parameters used for features of the space-time position of the SV or the observer, secondary values which allow improving the accuracy of the navigational and geodetic definitions can appear as evaluated elements. Values for defining a different type of errors are related to these elements, and also other constants which describe either the SV's movement or the conditions for transmitting electronic signals. /135

The practical use of different systems of parameters is due to the desire of obtaining the possibility of integrating differential equations of motion for a broad class of orbits; together with this, it is possible to greatly increase the accuracy of defining the parameters of motion for fixed conditions independent of the observer.

In the literature [16, 25], the choice of one system of parameters or another is considered only from the point of view of using it for predicting the SV's movement and the computational difficulties corresponding to this process. Moreover, it is evident that such a choice should be made, taking into account those properties which arise in the process of information processing for purposes of defining the selected set of parameters. The fact that the simplicity of algorithms and, in connection with this, the operation of defining and predicting the parameters of the SV's motion, in the final analysis, their accuracy, depends on the choice of the parameter system to a significant degree.

Not every system of parameters selected, used for describing the SV's motion in the entire range of their definition, results in a matrix of ACF signal second derivatives according to defined parameters which are sufficiently specified for solving an extreme problem by methods of successive approximations on contemporary computers with a completely defined capacity. Parameters in the form of components of the coordinates and velocity vector at a fixed moment of time easily lead to a concurrent solution. However, in solving a number of practical problems, the most suitable parameters for numerical and qualitative analysis are not the set of parameters which describe the initial conditions of motion in a geocentric rectangular or other equatorial or orbital systems of coordinates, but a system of osculating Keplerian elements and similar systems, since they give a more complete representation of the geometric characteristics of the orbit and its orientation in space. It should be noted that for examining the question of the distribution of the Earth's gravitational field in the space surrounding it, the set of osculating elements in the form of Keplerian parameters of orbit, evidently, is a unique system which offers the possibility of solving the problem posed [7] more simply. Moreover, the use of slowly changing parameters /136
which osculating Keplerian elements represent as the ephemeris of orbital radio-navigational or geodetic points for purposes of autonomously determining moving ground, surface or space objects draws special attention, since it allows significantly decreasing the size of the long-term storage of the on-board memory device and simplifies transmission of ephemeral information.

With small eccentricities in the elliptical orbit or insignificant angles of deviation in the mathematical relations which describe the differential equations of the SV's movement, the denominator approaches zero. This leads to the fact that accurate integration of differential equations of motion in the ranges of definition of the parameters indicated becomes complex or even impossible. As a consequence of this, defining the system of parameters selected will be accompanied by an increase in errors. Moreover, as we will show below, the correlation matrices of errors in defining the parameters of motion are characterized by the fact that in proceeding to the ranges of parameter definition mentioned above, the numerical values of the definition errors are significantly increased. Deterioration of the definition accuracy in the given case is due to the properties of the parameters' space as systems of reference accepted for the physical representation of the SV's position vector.

In order to solve the problem of making the parameters of motion more precise with the accuracy required for a broad class of orbits, the necessity arises in practice of using other elements of motion in place of those which produce many systems of parameters [6, 7, 21, 22, 28]. Although in much of the literature on celestial mechanics and the theory of a SV's flight, it is shown to be possible to integrate the differential equations of motion with the introduction of new systems of parameters, qualitative and quantitative analyses of changes in definition accuracy, due to the introduction of the new systems of parameters and changes in their values in the entire range of possible occurrences, could not be produced. Therefore, the purpose of the present chapter will be to examine the features for defining the SV's trajectories of motion by using different systems of parameters, especially, as we showed in the previous chapter, since the potential accuracy of their definition is a function of the composition of more precise parameters. Moreover, considering the question of the accuracy in evaluating the SV's position vector in different systems and the choice of a more accurate system of reference for determining the precise properties of the measurements systems is reasonable. /137

Considering that a linear value (a continuously changing range in the observation process) is the informative parameter of the signal received, it is useful to describe the exact properties of the measurement systems by the linear errors of the radius vector components and the velocity vector components or their equivalent values in a rectangular system of coordinates. Furthermore, in defining the parameters of motion, the specially chosen rectangular system of reference will be called initial. The choice of a rectangular coordinate system as initial is dictated by the fact that for defining the parameters of motion, additional transformations of the matrices of the ACF signal second derivatives are not required. Moreover, in a Cartesian rectangular system, the covariant and contravariant localized basis vectors coincide with the basis vectors of the system, whereas the localized basis vectors in orthogonal curvilinear systems of reference are functions of a point. A characteristic feature of rectangular systems is that the Cartesian coordinates /137 of any position vector and their differentials with respect to different systems are connected by linear functions. Linear coordinate transformation matrices and their differentials are identically equal and represent orthogonal rotation matrices. Therefore, for all rectangular systems of reference, the volume of the dispersion ellipsoid for defining the parameters of motion remains the same. In using other systems of reference, it is necessary to know their geometric properties and difference in comparison with rectangular systems.

5.2. Transforming Coordinates and Their Differentials. Transition Matrices

Any system of parameters of motion selected serves for describing the same principle of motion of a moving object. Therefore, naturally, a rigid unique interrelationship exists between the different systems which can be expressed by defined mathematical relations. The latter offers the possibility of transforming errors in defining the parameters during the transition from one system to another. However, prior to passing to an investigation of the accuracy in defining the velocity vector in different parameter areas, we should note some fundamentally important conditions which pertain to the differences /138
between transforming coordinates and their differentials and define the relations which describe these transformations.

Definition error transformations in using different systems of parameters (coordinates) as a system of reference for representing the position vector of the SV or the terrestial position of the observer are characterized by linear transformation matrices for the differentials of the position vector components, and not by coordinate transformation matrices. The connection between these is expressed by means of nonlinear functional dependencies whose form is defined in each concrete case by the composition of the parameters evaluated.

We will assume that the SV's position vector or that of the terrestial observer in an area of m-dimensional space can be given by means of different systems of independent parameters q and g; then each point $(q_1, q_2, \dots, q_m)$ of the m-dimensional space of the initial system of parameters q can be fixed corresponding to the ordered set m of real numbers $g_1, g_2, \dots, g_m$, which represent the value of the components of a finite parameter system g. The elements g_i of the position vector defined in the range of finite parameters g are connected with components g_j of initial system q by the relations

$$\left.\begin{array}{c} g_1 = g_1(q_1, q_2, \dots, q_m); \quad g_2 = g_2(q_1, q_2, \dots, q_m); \\ \dots; g_m = g_m(q_1, q_2, \dots, q_m), \end{array}\right| \qquad (6.2.1)$$

which in the position vector's definition range are everywhere equivalent, and the Jacobian of the transformation is not equal to zero:

$$\left|\frac{\partial(g_1, g_2, \ldots, g_m)}{\partial(q_1, q_2, \ldots, q_m)}\right| \neq 0. \tag{6.2.2}$$

It is characteristic that if equations (6.2.1) define the relation between the Cartesian systems of coordinates, generally speaking not rectangular, i.e., they describe the relation between the components of the observer's position vector with respect to different Cartesian systems of reference, in this case -- and only in this case -- all equations are linear and can be written by means of a linear transformation operator

$$g_{tr} = J_{tr} q_{tr}. \tag{6.2.3}$$

Transformation matrix J_{tr}, in the general case, is defined by the product of three factors, each of which is an orthogonal rotation matrix $R(\psi)$, which describes the rotation of the initial system of coordinates at angle ψ around one of its axes. The modulus of the Jacobian of the rotation matrix is equal to one. In this connection, since the elements of matrix J_{tr} are not functions of the components of vector g_{tr}, nor of the components of vector q_{tr}, for transforming the differentials of the coordinates, the relation will correctly be /139

$$dg_{tr} = J dq_{tr}. \tag{6.2.4}$$

In using Cartesian systems, the linearity of relations (6.2.1) is preserved, and where the defined position vector is complete, six-dimensional, i.e., more precise definition of not only the coordinates, but also the components of the velocity vector is effected. However, in this case it is better to refer to transformations of the position vector components and their differentials instead of to transformations of the coordinates and their differentials, since different systems of parameters can be used with the use of a defined spatial coordinate system for the characteristics of the position vector.

Moreover, the delimitation allows us to construct a sharp bound between the the transformations of parameters which characterize only the spatial position, and parameters used for describing the space-time position of moving objects. We should add that in the majority of cases, coordinate transformation matrices and their differentials are constituent elements of the formulas for transforming the components of a six-dimensional position vector and their differentials for representing a given vector in different parameter areas as systems of reference. Therefore, such a delimitation significantly facilitates further discussion of the material.

Transformations of the components of a six-dimensional position vector and their differentials with respect to different rectangular coordinate systems are described by quasi-diagonal matrices whose diagonal units are direction cosine matrices

$$\left\| \begin{matrix} g_{tr} \\ g_{tr} \end{matrix} \right\| = \left\| \begin{matrix} J_{tr} & 0 \\ 0 & J_{tr} \end{matrix} \right\| \left\| \begin{matrix} q_{tr} \\ q_{tr} \end{matrix} \right\|; \qquad \left\| \begin{matrix} dg_{tr} \\ dg_{tr} \end{matrix} \right\| = \left\| \begin{matrix} J_{tr} & 0 \\ 0 & J_{tr} \end{matrix} \right\| \left\| \begin{matrix} dq_{tr} \\ dq_{tr} \end{matrix} \right\|, \tag{6.2.5}$$

where q_{tr} and g_{tr} are vectors of the velocity components in the rectangular coordinate systems studied; dq_{tr} and dg_{tr} are the vectors of the velocity components' differentials.

Inverse transformations in the entire area of possible occurrence of the position vector are allowed for formulas (6.2.3)-(6.2.5). In this connection, the linear operator of the inverse transformation is identical to the transposed value of the direct transformation. /140

In the general case, expressions (6.2.1) are non-linear with respect to components q_j by relations:

$$g = g(q),$$

whose complexity in selected system g is a function of the composition of parameters q, used as the initial system's components.

Simpler formulas for transforming the coordinates are the relations which define the relation between the components of Cartesian rectangular and universal systems whose origin and basic planes coincide.

Thus, for a cylindrical coordinate system with components $q_c^T = ||\rho\ \lambda_c\ z_c||$, this transformation takes the form of

$$g_{tr} = R_z(-\lambda_c) Q_c, \qquad (6.2.6)$$

where $g_{tr}^T = ||x\ y\ z||$ is the position vector in a rectangular system of coordinates; $Q_c^T = ||\rho\ 0\ z_c||$ is the vector defined by the linear components of the cylindrical system of reference and which describes one of the coordinates of the lines of the observer's position; $R_z(-\lambda_c)$ is a matrix which describes the transformation of rectangular coordinates in rotation. The positive value of argument λ_c describes counter-clockwise rotation.

For a spherical system with components $q_c^T = ||r_K \lambda_s\ \phi||$, the transformation takes the form of

$$g_{tr} = R_z(-\lambda_s) R_y(-\phi) Q_s, \qquad (6.2.7)$$

where $R_z(-\lambda_s)$ and $R_y(-\phi)$ are rotation matrices; $Q_s^T = ||r_K\ 0\ 0||$ is the linear coordinate of a spherical system of reference.

The relationship between the coordinates of the Cartesian rectangular and universal geodetic systems of reference with components $g_g^T = ||H\ L\ B_g||$ is defined by the relation

$$g_{tr} = \mathbf{R}_z(-L)\,\mathbf{R}_y(-B_g)\,\mathbf{Q}_g + \mathbf{Q}_0, \qquad (6.2.8)$$

which can be represented as a transformation of the coordinates of some quasi-spherical system to rectangular systems. It is characteristic that the position of the origin and basic plane of the quasi-spherical system does not remain constant. For the parameters accepted (of semimajor axis a_3 and eccentricity e_3) of the reference ellipsoid with respect to whose plane altitude H is measured, the position of the origin and basic plane of the quasi-spherical system of reference changes with a change in geodetic latitude B_g. In this connection, the basic plane of the quasi-spherical system is shifted parallel to plane OXY of the rectangular coordinate system, and the origin is along axis OZ. This shift is described by vector $Q_0^T = ||0\ 0\ -Ne_3^2 \sin B_g||$. Its maximum value is equal to $\pm a_3 e_3^2/\sqrt{1-e_3^2}$. Linear coordinate $Q_g^T = ||N + H\ 0\ 0||$ represents the sum of the altitude H and the radius of curvature $N = a_3/\sqrt{1-e_3^2 \sin^2 B}$ in the plane of the reference ellipsoid along the first vertical at the observation /141 point.

Relation (6.2.8) can be replaced by the following expression which is equivalent to it:

$$g_{tr} = R_z(-L)R_y(-B_g)Q_{g0},$$

where $Q_{g0}^T = ||N(1-e_3^2 \sin^2 B_g) + H0 - Ne_3^2 \sin B_g \cos B_g||$.

Relations (6.2.6)-(6.2.8) can serve as the basis for obtaining both the formulas of coordinate transformation for the transition from different curvilinear to Cartesian systems of reference by means of supplementary linear operators, and of linear operators for transforming the coordinate differentials.

Thus, in examining cylindrical and spherical orbital systems of reference whose basic planes are combined with the plane of the osculating ellipsis, and the polar axes at the moment of osculation, coincide with the sense at the ascending node, formulas for transforming the coordinates are defined by the following relations:

$$\left.\begin{aligned} g_{tr} &= R_z(-\Omega)R_x(-i)R_z(-u)Q_c \\ g_{tr} &= R_z(-\Omega)R_x(-i)R_z(-u)R_y(-\varphi_0)Q_s \end{aligned}\right\} \quad (6.2.9)$$

in which the arguments of the rotation matrices represent the values: u - the latitude argument; i - the orbital plane deviation; Ω - the longitude of the ascending node; ϕ - the latitude with respect to the orbital plane.

Formulas for transforming the differentials of the coordinates for curvilinear systems of reference are defined by the expressions: /142

$$\left.\begin{aligned} dg_{tr} &= R_z(-\lambda_c)W_c dq_c \\ dg_{tr} &= R_z(-\lambda_c)R_y(-\varphi)W_s dq_s \\ dg_{tr} &= R_z(-L)R_y(-B_g)W_g dq_g \end{aligned}\right\}$$

(6.2.10)

The differentials are interlinked by means of the linear operators

$$\left.\begin{aligned} J_c &= R_z(-\lambda_c)\,W_c = R_c W_c \\ J_s &= R_z(-\lambda_s)\,R_y(-\varphi)\,W_s = R_s W_s \\ J_g &= R_z(-L)\,R_y(-B)\,W_g = R_g W_g \end{aligned}\right\} \tag{6.2.11}$$

in which are included matrices W_c, W_s and W_g), which directly describe the transformation of coordinate differentials of the coordinate systems examined, in contrast to the formulas for transforming the differentials of Cartesian systems together with orthogonal matrices of rotation R_c, R_s, and R_g. In the special case of converting from coordinate errors in curvilinear (cylindrical, spherical and geodetic) systems to errors in the components of Cartesian rectangular systems, the direct transformation matrices W_c, W_s, and W_g describe the transformation of angular parameter errors in linear systems and are defined by the following relations:

$$\left.\begin{aligned} W_c &= \begin{Vmatrix} 1 & 0 & 0 \\ 0 & \rho & 0 \\ 0 & 0 & 1 \end{Vmatrix}, \quad W_s = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & r_s\cos\varphi & 0 \\ 0 & 0 & r_s \end{Vmatrix}, \\ W_g &= \begin{Vmatrix} 1 & 0 & 0 \\ 0 & (N+H)\cos B & 0 \\ 0 & 0 & N_1+H \end{Vmatrix}, \end{aligned}\right\} \tag{6.2.13}$$

where $$N_1 = \frac{N(1-e_3^2)}{(1-e_3^2\sin^2 B_r)}.$$

Inverse transformations in the entire range of possible coordinate assignment with the exception of the specific points in which matrices W_c, W_s, and W_g become specific, are permitted in formulas (6.2.10). For this reason, with a constant value of the linear errors in relation to approaching singular points, errors in the angular components of vectors dq_c, dq_s and dq_g increase, which is seen from the following relations:

$$
\left.\begin{aligned}
d\lambda_c &= \frac{1}{\rho}(dy\cos\lambda_c - dx\sin\lambda_c),\\
d\lambda_s &= \frac{1}{r_s\cos\varphi}(dy\cos\lambda_s - dx\sin\lambda_s),\\
d\varphi &= \frac{1}{r}[dz\cos\varphi - (dx\cos\lambda_s + dy\sin\lambda_s)\sin\varphi],\\
dL &= \frac{1}{(N+H)\cos B_g}(dy\cos L - dx\sin L),\\
dB_g &= \frac{1}{N_1+H}[dz\cos B_g - (dx\cos L + dy\sin L)\sin B_g].
\end{aligned}\right\} \quad (6.2.13)
$$

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Transformations of the position vector in converting from a curvilinear to a rectangular system of coordinates in a partitioned matrix form of notation are defined by the relation

$$
\left\|\frac{\mathbf{g}_{tr}}{\mathbf{g}_{tr}}\right\| = \left\|\begin{matrix}\mathbf{R} & 0\\ 0 & \mathbf{J}\end{matrix}\right\| \left\|\frac{\mathbf{Q}}{\mathbf{q}}\right\|, \quad (6.2.14)
$$

in which under matrices R, J, Q and q, one of the sets of matrices R_c, J_c, Q_c and q_c or R_s, J_s, Q_s and q_s or R_g, J_g, Q_g and q_g, depending on which of the curvilinear coordinate systems is used, should be understood. Vector q is described by the velocity components of a curvilinear system. Thus, in transforming the coordinates of a spherical system, the vector in question is determined by the function $q_s^T = ||r_K \ \lambda_s \ \phi||$.

The formulas for transforming the differentials of position vector components in converting from curvilinear systems of reference to rectangular systems can be defined in a general form by the expression

$$
\left\|\frac{d\mathbf{g}_{tr}}{d\dot{\mathbf{g}}_{tr}}\right\| = \left\|\begin{matrix}\mathbf{R} & 0\\ 0 & \mathbf{R}\end{matrix}\right\| \left\|\begin{matrix}\mathbf{W} & 0\\ \mathbf{V} & \mathbf{W}\end{matrix}\right\| \left\|\frac{d\mathbf{q}}{d\dot{\mathbf{q}}}\right\|, \quad (6.2.15)
$$

in which the outer diagonal block V (V_c, V_s and V_g, corresponding to cylindrical, spherical and geodetic systems of reference) of the matrix of direct differential transformations describes the transformation of errors in angular velocities into linear velocities, resulting from errors in the coordinate components of vectors dg_g, dg_c and dg_s. Matrix R has the same sense as in relation (6.2.14). Matrix W is defined by one of the expressions of (6.2.12) with regard to the curvilinear system used. Transformations of the first two factors of the right part of expression (6.2.15) represent none other than the transition matrix whose diagonal blocks are defined by relations (6.2.11). The outer diagonal blocks (V_c, V_s, and V_g) of the direct transformation matrix of differentials which simultaneously are the outer diagonal blocks of the transition matrix, can be defined by means of the expressions /144

$$V_c = \begin{Vmatrix} 0 & -\rho\dot{\lambda}_c & 0 \\ \dot{\lambda}_c & \dot{\rho} & 0 \\ 0 & 0 & 0 \end{Vmatrix},$$

$$V_g = \begin{Vmatrix} 0 & -r_s\dot{\lambda}_c\cos^2\varphi & -r_s\dot{\varphi} \\ \dot{\lambda}_s\cos\varphi & \dot{r}_s\cos\varphi - r_s\dot{\varphi}\sin\varphi & -r_s\dot{\lambda}_s\sin\varphi \\ \dot{\varphi} & r_s\dot{\lambda}_s\sin\varphi\cos\varphi & \dot{r} \end{Vmatrix},$$

$$V_r = \begin{Vmatrix} 0 & -(N+H)\dot{L}\cos^2 B_g & -(N_1+H)\dot{B}_g \\ \dot{L}\cos B_g & \dot{H}\cos B_g-(N_1+H)\dot{B}_g\sin B_g & -(N_1+H)\dot{L}\sin B_g \\ \dot{B}_g & (N+H)\dot{L}\sin B_g\cos B_g & \dot{H}+N_2\dot{B}_g \end{Vmatrix},$$

(6.2.16)

where $N_2 = 3\cdot N_1 e_3^2 \sin B_g \cos B_g/(1-e_3^2\sin^2 B_g)$

The inverse transformation of the position vector component differentials can be represented in a general form by the function

$$\begin{Vmatrix} dq \\ d\dot{q} \end{Vmatrix} = \begin{Vmatrix} W^{-1} & 0 \\ U & W^{-1} \end{Vmatrix} \begin{Vmatrix} R^T & 0 \\ 0 & R^T \end{Vmatrix} \begin{Vmatrix} dg_{tr} \\ d\dot{g}_{tr} \end{Vmatrix}, \qquad (6.2.17)$$

in which the matrix element U (U_c, U_s and U_g, corresponding to cylindrical, spherical and geodetic systems of reference) is defined by one of the relations

$$U_c = \begin{Vmatrix} 0 & \dot{\lambda}_c & 0 \\ -\frac{\dot{\lambda}_c}{\rho} & -\frac{\dot{\rho}}{\rho^2} & 0 \\ 0 & 0 & 0 \end{Vmatrix},$$

$$
\mathbf{U}_{c\varphi}=\left\|\begin{matrix} 0 & \dot\lambda_s\cos\varphi & \dot\varphi \\ -\dfrac{\dot\lambda_s}{r_s} & \dfrac{r_s\dot\varphi\sin\varphi-\dot r_s\cos\varphi}{r_s^2\cos^2\varphi} & \dfrac{\dot\lambda_s}{r_s}\operatorname{tg}\varphi \\ -\dfrac{\dot\varphi}{r_s} & -\dfrac{\dot\lambda_s}{r_s}\sin\varphi & -\dfrac{\dot r_s}{r_s^2} \end{matrix}\right\|,
$$

(6.2.18)

$$
\mathbf{U}_g=\left\|\begin{matrix} 0 & \dot L\cos B_g & \dot B_g \\ -\dfrac{\dot L}{N+H} & \dfrac{(N_1+H)\dot B_g\sin B_g-\dot H\cos B_g}{(N+H)^2\cos^2 B_g} & \dfrac{\dot L}{N+H}\operatorname{tg}B_g \\ -\dfrac{\dot B_g}{N_1+H} & -\dfrac{\dot L}{N_1+H}\sin B_g & -\dfrac{\dot H+N_2\dot B_g}{(N_1+H)^2} \end{matrix}\right\|.
$$

Matrix W^{-1} is inverse with respect to matrix W of expression (6.2.15). Therefore, relation (6.2.17) is valid within the range of the SV's position vector definition in which matrix W is ordinary.

Using relations (6.2.15) and (6.2.17), it is possible to define the relation between the differentials of the position vector components in the transition from the k^{th} to the j^{th} curvilinear system of reference. This relation is described by the expression

$$
\left\|\begin{matrix} d\mathbf{q}_j \\ d\dot{\mathbf{q}}_j \end{matrix}\right\|=\left\|\begin{matrix} \mathbf{W}_j^{-1} & 0 \\ \mathbf{U}_j & \mathbf{W}_j^{-1} \end{matrix}\right\|\left\|\begin{matrix} \mathbf{R}_j^T\mathbf{R}_k & 0 \\ 0 & \mathbf{R}_j^T\mathbf{R}_k \end{matrix}\right\|\times
$$
$$
\times\left\|\begin{matrix} \mathbf{W}_k & 0 \\ \mathbf{V}_k & \mathbf{W}_k \end{matrix}\right\|\left\|\begin{matrix} d\mathbf{q}_k \\ d\dot{\mathbf{q}}_k \end{matrix}\right\|, \qquad (6.2.19)
$$

which is valid if the polar axes of both systems coincide. In the transition from a spherical to a cylindrical system, relation (6.2.19) is significantly simplified, since the product of

$R_S^T R_S$ is identically equal to rotation matrix $R_\nu(-\phi)$.

Formulas (6.2.15), (6.2.17) and (6.2.19) describe the transformation of the initial conditions of motion with respect to different coordinate systems whose classification and enumeration can be found in [26].

Let us proceed to systems of parameters which include Keplerian and similar orbital elements. In this respect, we should consider that in all cases where Keplerian and similar parameters appear as components of the SV's position vector, /146 as a rule, the coordinates and components of the velocity vector are used as the initial coordinates for all methods of navigational and geodetic definitions in an inertial geocentric rectangular system of reference [7]. Therefore, it is necessary to obtain the relations which connect the initial conditions of motion in an inertial rectangular coordinate system and their differentials with the corresponding Keplerian or similar elements of orbit. These relations must be suitable for applying different operations of matrix calculus.

Furthermore, it will be assumed that all systems of parameters of SV motion examined describe its position in m-dimensional spaces at some moment of time t_0. Without losing generality, we can assume that at moment t_0 the SV's movement occurs according to a purely Keplerian orbit. In this connection, the trajectories of motion are represented by a plane curve for defining the SV's position in which other instantaneous arguments -- true υ and eccentric E anomalies, average latitude $M + \omega$, etc., can be used instead of time t. Usually, the origin of the instantaneous argument is related to the SV's moment of passage of the perigee or the ascending node. If we take into account that some Keplerian and similar orbital elements are also directly connected with the reference point of an instantaneous variable, then in defining the interrelationship between the initial conditions of motion in rectangular coordinate system OXYZ (Fig. 6.1) and the Keplerian or similar parameters, it is necessary to introduce some intermediate systems of reference. Thus, in using Keplerian parameters of orbit which require using an instantaneous variable, whose origin coincides with the moment of the SV's passing the perigee, it is expedient to examine a geocentric orbital rectangular system of coordinates $OX_1Y_1Z_1$, whose axis OX_1 coincides with the direction on the perigee, as an intermediate system. At the same time, three angular elements i, ω and Ω of a Keplerian parameter system which describes the orientation of the orbit in space, are simultaneously elements of the direction cosines between the axes of coordinate systems $OX_1Y_1Z_1$ and OXYZ (Fig. 6.1). The three other orbital parameters a, e

and M_0 define the conic section regardless of the coordinate system selected and, describing the space-time position of the SV in orbit, give a comprehensive idea of its motion in the orbital system of coordinates $OX_1Y_1Z_1$.

In examining systems of parameters similar to Keplerian systems and canonical parameters of motion, taking into consideration that they all have a single functional dependence relative to Keplerian elements of orbit, a geocentric system of rectangular coordinates must be considered as the initial system, and a system of Keplerian elements i, ω, Ω, a, e, M_0 as the intermediate system. /147

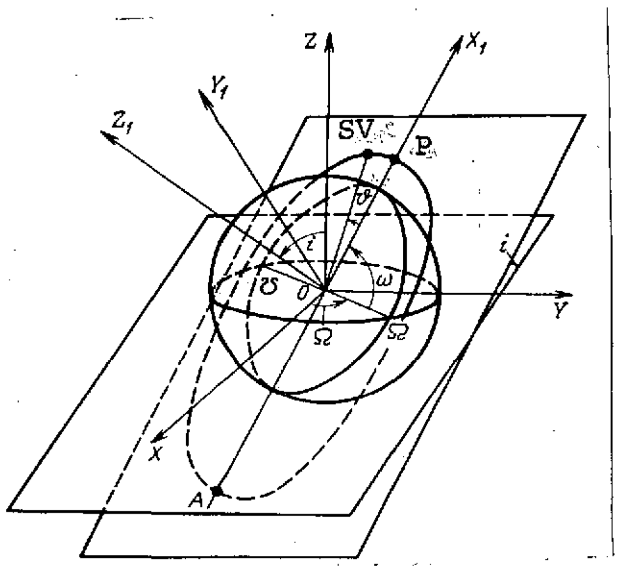

Fig. 6.1. Geocentric equatorial and orbital systems of coordinates.

For defining the interrelationship of the components of the SV's position vector in the area of initial conditions of motion $g^t = ||x\ y\ z\ \dot{x}\ \dot{y}\ \dot{z}||$ in an inertial rectangular system of coordinates OXYZ and in the area of Keplerian elements of orbit, $q^T = ||i\ \omega\ \Omega\ a\ e\ M_0||$, we will represent vector g by the following expression (Fig. 6.1):

$$g = GHSg_1 \qquad (6.2.20)$$

where $g_1^T = ||x_1\ y_1\ z_1\ \dot{x}_1\ \dot{y}_1\ \dot{z}_1||$ is the SV's position vector in the region of initial conditions in a geocentric-orbital rectangular system $OX_1Y_1Z_1$; G, H and S are matrices of a quasi-diagonal form whose diagonal blocks are rotation matrices

$R_z(-\Omega)$, $R_x(-i)$ and $R_z(-\omega)$, respectively.

The elements of matrices G, H and S are defined only by the Keplerian parameters of orbit which describe its orientation in the area of a system of coordinates OXYZ. Vector g_1 is a function of the intraplanar Keplerian elements of orbit and the instantaneous argument selected. The representation of vector g_1 is significantly simplified if a value of eccentric E or true υ is chosen as the argument. Taking this into account, we can describe vector g_1 by an expression which is a function of the eccentric anomaly: /148

$$g_1 = \left\| \begin{array}{c} a(\cos E - e) \\ a\sqrt{1-e^2}\sin E \\ 0 \\ -\sqrt{\frac{\mu}{a}}\,\frac{\sin E}{1-e\cos E} \\ \sqrt{\frac{\mu}{a}}\,\frac{\sqrt{1-e^2}\cos E}{1-e\cos E} \\ 0 \end{array} \right\|, \tag{6.2.21}$$

where a is the semimajor axis of the ellipse; e is the eccentricity of orbit; μ is a coefficient equal to the product of the gravitational constant in the Earth's mass, and also a function of the true anomaly:

$$g_1 = \left\| \begin{array}{c} \frac{a(1-e^2)\cos\vartheta}{1+e\cos\vartheta} \\ \frac{a(1-e^2)\sin\vartheta}{1+e\cos\vartheta} \\ 0 \\ -\sqrt{\frac{\mu}{a}}\,\frac{\sin\vartheta}{\sqrt{1-e^2}} \\ \sqrt{\frac{\mu}{a}}\,\frac{e+\cos\vartheta}{\sqrt{1-e^2}} \\ 0 \end{array} \right\|. \tag{6.2.22}$$

Thus, expression (6.2.20) defines the functional dependence of position vector g on the components of vector q. For finding the formula for transforming the differentials of position vector components g and q, it is necessary to define the transition matrix, the P-matrix, of partial derivatives of coordinate components and the velocity vector in system OXYZ by Keplerian parameters of orbit. It is possible to represent the indicated matrix as a derivative of position vector g according to position vector q [7, 14]: P = δg/δq. In a form more convenient for study, the transition matrix can be written as /149

$$\mathbf{P}=\left\| \frac{\partial \mathbf{g}}{\partial i} \ \frac{\partial \mathbf{g}}{\partial \omega} \ \frac{\partial \mathbf{g}}{\partial \Omega} \ \frac{\partial \mathbf{g}}{\partial a} \ \frac{\partial \mathbf{g}}{\partial e} \ \frac{\partial \mathbf{g}}{\partial M_0} \right\|, \quad (6.2.23)$$

in which each element $P_{\cdot j}$, representing a six-dimensional vector (matrix column), taking into account relation (6.2.20), is defined by the product of several matrices.

Thus, the first three elements of expression (6.2.23) are defined by the following relations:

$$\mathbf{P}_{\cdot 1}=\mathbf{G}\frac{\partial \mathbf{H}}{\partial i}\mathbf{S}\mathbf{g}_1, \quad \mathbf{P}_{\cdot 2}=\mathbf{G}\mathbf{H}\frac{\partial \mathbf{S}}{\partial \omega}\mathbf{g}_1, \quad \mathbf{P}_{\cdot 3}=\frac{\partial \mathbf{G}}{\partial \Omega}\mathbf{H}\mathbf{S}\mathbf{g}_1, \quad (6.2.24)$$

which can be easily calculated if we consider that the first three factors of matrix P are quasi-diagonal matrices, and the differentiation operations, which do not change the structure, result in their simplification.

The last three elements of matrix P, defined by expression (6.2.23), can be represented by the relations

$$\mathbf{P}_{\cdot 4}=\mathbf{G}\mathbf{H}\mathbf{S}\frac{\partial \mathbf{g}_1}{\partial a}, \quad \mathbf{P}_{\cdot 5}=\mathbf{G}\mathbf{H}\mathbf{S}\frac{\partial \mathbf{g}_1}{\partial e}, \quad \mathbf{P}_{\cdot 6}=\mathbf{G}\mathbf{H}\mathbf{S}\frac{\partial \mathbf{g}_1}{\partial M_0}. \quad (6.2.25)$$

In this connection, allowing for expression (6.2.21), the derivative of position vector g_1 by scalar a is defined by the function

$$\frac{\partial g_1}{\partial a} = \left\| \begin{matrix} \cos E - e \\ \sqrt{1-e^2} \sin E \\ 0 \\ \frac{1}{2a}\sqrt{\frac{\mu}{a}} \frac{\sin E}{1-e\cos E} \\ -\frac{1}{2a}\sqrt{\frac{\mu}{a}} \frac{\sqrt{1-e^2}\cos E}{1-e\cos E} \\ 0 \end{matrix} \right\|. \tag{6.2.26}$$

Vector g_1 is a function of the eccentricity, both directly and through the eccentric anomaly. Therefore, for finding the derivative $\delta g_1/de$, it is necessary to define the partial differential of position vector g_1 by scalar e:

$$d_e g_1 = \frac{\partial g_1}{\partial_e} de + \frac{\partial g_1}{\partial E} \frac{\partial E}{\partial e} de, \tag{6.2.27}$$

whose component elements are a partial derivative of the eccentric anomaly with respect to the eccentricity. The latter can be defined by differentiating a Keplerian equation:

$$\begin{gathered} E - e \sin E = M, \\ \frac{\partial E}{\partial e} - \sin E - e\frac{\partial E}{\partial e} \cos E = 0. \end{gathered} \tag{6.2.28}$$

Whereby we have

$$\frac{\partial E}{\partial e} = \frac{\sin E}{1 - e\cos E}. \tag{6.2.29}$$

$$\frac{\partial \mathbf{g}_1}{\partial e} = \left\| \begin{matrix} a\dfrac{(\cos E + e)\cos E - 2}{1 - e\cos E} \\ a\dfrac{(\cos E - e)\sin E}{\sqrt{1-e^2}\,(1 - e\cos E)} \\ 0 \\ -\sqrt{\dfrac{\mu}{a}}\,\dfrac{(2 - e\cos E)\cos E - e}{(1 - e\cos E)^3}\sin E \\ \sqrt{\dfrac{\mu}{a}}\,\dfrac{(2\cos E - e\cos^2 E - e)\cos E + e^2 - 1}{\sqrt{1-e^2}\,(1 - e\cos E)^3} \\ 0 \end{matrix} \right\|. \tag{6.2.30}$$

The partial derivative of position vector g_1 according to mean anomaly M_0 is expressed by the product of two factors, g_1/dE and dE/dM_0. The latter can be defined by means of a differentiation corresponding to expressions (6.2.21) and (6.2.28). Therefore, the relation is direct

$$\frac{\partial \mathbf{g}_1}{\partial M_0} = \left\| \begin{matrix} -a\dfrac{\sin E}{1 - e\cos E} \\ a\dfrac{\sqrt{1-e^2}\cos E}{1 - e\cos E} \\ 0 \\ -\sqrt{\dfrac{\mu}{a}}\,\dfrac{\cos E - e}{(1 - e\cos E)^3} \\ -\sqrt{\dfrac{\mu}{a}}\,\dfrac{\sqrt{1-e^2}\sin E}{(1 - e\cos E)^3} \\ 0 \end{matrix} \right\|. \tag{6.2.31}$$

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Representing transition matrix P in the form of a partial derivative of position vector g, defined by the product of the matrices according to vector q, whose components appear as Keplerian parameters, we can easily define any of the rows of the matrix and any of its elements. Thus, the k^{th} row of matrix P can be expressed by the following function:

$$\mathbf{P}_{k\cdot} = \frac{\partial g_k}{\partial \mathbf{q}} = \frac{\partial}{\partial \mathbf{q}} (\mathbf{G}_{k\cdot} \mathbf{HS} \mathbf{g}_1), \tag{6.2.32}$$

where G_k is the k^{th} row of matrix G. Taking into account the principle of differentiating scalar quantity g_k according to vector] [14], expression (6.2.32) can be reduced to the relation

$$\mathbf{P}_{k\cdot} = \left\| \frac{\partial g_k}{\partial i} \; \frac{\partial g_k}{\partial \omega} \; \frac{\partial g_k}{\partial \Omega} \; \frac{\partial g_k}{\partial a} \; \frac{\partial g_k}{\partial e} \; \frac{\partial g_k}{\partial M_0} \right\|, \tag{6.2.33}$$

whose elements, according to the form of notation, can be represented by formulas similar to the functions which define matrix column $P_{\cdot j}$ of expression (6.2.23). However, their essential difference is that $P_{\cdot j}$ are vectors, whereas the elements of matrix $P_{k\cdot}$ are scalar values, functionally dependent on the components of vector q. Another distinctive feature is that matrix row G_k, or its derivative $\delta G_k/\delta\Omega$, stands in place of matrix G or its derivative with respect to angle Ω. In corroboration of the above, we will write the relations which define the elements of matrix row $P_{k\cdot}$:

$$\left.\begin{aligned}
p_{k1} &= \mathbf{G}_{k\cdot} \frac{\partial \mathbf{H}}{\partial i} \mathbf{S} \mathbf{g}_1; \quad p_{k2} = \mathbf{G}_{k\cdot} \mathbf{H} \frac{\partial \mathbf{S}}{\partial \omega} \mathbf{g}_1, \\
p_{k3} &= \frac{\partial \mathbf{G}_{k\cdot}}{\partial \Omega} \mathbf{HS} \mathbf{g}_1, \quad p_{k4} = \mathbf{G}_{k\cdot} \mathbf{HS} \frac{\partial \mathbf{g}_1}{\partial a}, \\
p_{k5} &= \mathbf{G}_{k\cdot} \mathbf{HS} \frac{\partial \mathbf{g}_1}{\partial e}, \quad p_{k6} = \mathbf{G}_{k\cdot} \mathbf{HS} \frac{\partial \mathbf{g}_1}{\partial M_0}.
\end{aligned}\right\} \tag{6.2.34}$$

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Expressions (6.2.34), which allow defining any of the elements of transition matrix P, show that its elements define the complex functions of the components of vector q. The appearance of the elements of the differential direct transformation matrix as a function of the components of the initial system of parameters is characteristic of all non-orthogonal matrices, including the matrices described by expressions (6.2.15) and (6.2.19). However, the complexity of this function is defined both by the components of the initial and terminal systems of parameters, which can be seen in the example which compares relations (6.2.15) and (6.2.19). The property referred to of elements of nonorthogonal transformation matrices leads to the fact that their determinants also are a function of some components of the initial system of parameters. Therefore, the magnitude of the determinants does not remain constant with a change in the range of parameter definition; at singular points, it is equal to zero.

Using expression (6.2.20), and also the general, and some special, properties of orthogonal matrices, the properties of the derivatives of orthogonal matrices according to the angular arguments and their derivatives, it is possible to represent the formula for transforming the differentials of a six-dimensional Keplerian parameter vector in initial conditions of motion in a rectangular system of reference OXYZ by the relation

$$\left\| \begin{matrix} d\mathbf{g}_0 \\ d\dot{\mathbf{g}}_0 \end{matrix} \right\| = \left\| \begin{matrix} \mathbf{R}_S & 0 \\ 0 & \mathbf{R}_S \end{matrix} \right\| \left\| \begin{matrix} \mathbf{W}_1 & \mathbf{W}_3 \\ \mathbf{W}_2 & \mathbf{W}_4 \end{matrix} \right\| \left\| \begin{matrix} d\mathbf{q}_1 \\ d\mathbf{q}_2 \end{matrix} \right\|, \quad (6.2.35)$$

where $dg_0^T = ||dx\ dy\ dz||$; $d\dot{g}_0^T = ||d\dot{x}\ d\dot{y}\ d\dot{z}||$ are differentials of the coordinate and velocity components of initial conditions of motion g; $dq_1^T = ||di\ d\omega\ d\Omega||$; $dq_2^T = ||da\ de\ dM_0||$ are differentials of the angular and intraorbital Keplerian elements; $R_K = R_z(-\Omega)R_x(-i)R_z(-\omega)$ is an orthogonal coordinate transformation matrix in going from system $OX_1Y_1Z_1$ to system OXYZ; W_1, W_2, W_3, W_4 are blocks of matrix W_K for the direct transformation of differentials of Keplerian parameters into differentials of linear coordinates and components of the velocity vector. /153

Matrices W_3 and W_4, representing coordinate and velocity component derivatives of the vector of initial conditions of motion g_1 in orbital system of reference $OX_1Y_1Z_1$, according to the intraorbital Keplerian parameters a, e and M_0,

$$\mathbf{W}_3 = \frac{\partial (x_1\, y_1\, z_1)}{\partial (u\, e\, M_0)}, \quad \mathbf{W}_4 = \frac{\partial (\dot{x}_1\, \dot{y}_1\, \dot{z}_1)}{\partial (a\, e\, M_0)}, \quad (6.2.36)$$

are defined by the corresponding components of expressions (6.2.26), (6.2.30), and (6.2.31). If a true anomaly is used as the instantaneous variable, then the matrices shown are expressed by the functions

$$\mathbf{W}_3 = \left\| \begin{matrix} \frac{(1-e^2)\cos\vartheta}{h} & -\frac{a(h+\sin^2\vartheta)}{h} & -\frac{a\sin\vartheta}{\sqrt{1-e^2}} \\ \frac{(1-e^2)\sin\vartheta}{h} & \frac{a\sin 2\vartheta}{2h} & \frac{a(e+\cos\vartheta)}{\sqrt{1-e^2}} \\ 0 & 0 & 0 \end{matrix} \right\|,$$

(6.2.37)

$$\mathbf{W}_4 = \left\| \begin{matrix} \frac{\sin\vartheta}{2a\sqrt{1-e^2}} & -\frac{e+(1+h)\cos\vartheta}{(1-e^2)^{3/2}}\sin\vartheta & -\frac{h^2\cos\vartheta}{(1-e^2)^2} \\ -\frac{e+\cos\vartheta}{2a\sqrt{1-e^2}} & \frac{h\cos^2\vartheta-\sin^2\vartheta}{(1-e^2)^{3/2}} & -\frac{h^2\sin\vartheta}{(1-e^2)^2} \\ 0 & 0 & 0 \end{matrix} \right\| \sqrt{\frac{\mu}{a}},$$

where $h=1+e\cos\nu$. In this connection, the first, second and third columns of matrices W_3 and W_4 are identically equal to relations (6.2.26), (6.2.30) and (6.2.31), if in the latter the eccentric anomaly changes to a true anomaly.

Matrices W_1 and W_2 can be represented by expressions

$$\mathbf{W}_1 = \left\| \begin{matrix} 0 & -\sin\vartheta & -\sin\vartheta\cos i \\ 0 & \cos\vartheta & \cos\vartheta\cos i \\ \sin(\omega+\vartheta) & 0 & -\cos(\omega+\vartheta) \end{matrix} \right\| \cdot \frac{a(1-e^2)}{1+e\cos\vartheta},$$

$$\mathbf{W}_2 = \left\| \begin{matrix} 0 & -(e+\cos\vartheta) & -(e+\cos\vartheta)\cos i \\ 0 & -\sin\vartheta & -\sin\vartheta\cos i \\ \cos(\omega+\vartheta)+e\cos\omega & 0 & \sin(\omega+\vartheta)+e\sin\omega \end{matrix} \right\| \times$$

$$\times\sqrt{\frac{\mu}{a(1-e^2)}},$$

(6.2.38) /154

whose elements are defined by the coordinate or velocity components of vector g_1, by the angular distance of the perigee and the deviation of the orbit.

A characteristic of the differential direct transformation matrix is that its elements are not functions of the longitude of the ascending node and are defined by intraorbital elements a, e, M_0 and two angular parameters, i and ω.

Expression (6.2.35) shows that the transition matrix R, representing a linear transformation operator, is defined by the product

$$P = GHSW_K, \tag{6.2.39}$$

one of whose factors is the direct differential transformation matrix W_K.

Transition matrix P describes the transformation of position vector component differentials when Keplerian elements of orbit are used as the initial system of parameters. If the initial conditions of motion in a rectangular system of reference OXYZ appear as the initial system of parameters, then the differential transformation is defined by the expression

$$dq = W_K^{-1}S^T H^T G^T dg, \tag{6.2.40}$$

which is direct in all regions of parameter definition with the exception of the special points in which matrix W_K becomes singular.

The transformations examined show that the connection between errors in the components of the SV's position vector or the terrestial observer, with respect to the different multidimensional parameter spaces, is defined by a linear operator of a differential transformation in the transition from one system of reference to another. Subsequently, the differential transformation matrix which describes the transformation of errors in transition, in contrast to the coordinate transformation matrix, will be called the transition matrix. The transition matrices which connect the differentials of parameters of motion in different systems of reference with unequal dimensions of physical coordinates, together with orthogonal matrices, also include nonorthogonal matrices which describe the direct differential transformations of parameters with non-identical dimensions. Being linear operators, transition matrices describe the transformation of coordinates in the case -- and only in the case -- where, first, the coordinate systems examined are Cartesian (generally speaking, optionally rectangular) and second, the position vector's components only describe the location of the SV or the observer in the chosen system of reference. In describing the transformation of errors in the transition from one region of parameters to another and indicating the special features of these areas, the transition matrices play /155 in problems of defining the parameters of motion.

5.3. A Quantitative Approach to Evaluating Properties for Defining Different Systems of Parameters

In evaluating the accuracy and definition of a velocity vector in the field of the signal received with regularly changing parameters, operations for finding the first and second ACF signal derivatives according to the components of the vector of evaluated elements at the point of their a priori knowledge, and solution of equations (3.1.7) and (3.1.8) are related to the significant operations of time-space filtration. Correlation matrix B_g for defining corrections to precise parameters g in the case of the absence of matrix B_{ga} of errors in a priori data coincide with correlation matrix B_{gm}, for measurements, and with accuracy to constant factors, is numerically equal to the inverse matrix of the ACF signal second derivatives. Thus, for a signal with regularly changing amplitude and fluctuating initial phase, the corrections vector Δg and correlation matrix B_g are defined by the relations

$$\Delta \mathbf{g} = \frac{2}{N_0} \mathbf{B}_g \sum_{k=1}^{N} \left[\mathbf{Z}'_{gk}(\mathbf{g}_a) - \frac{1}{2} \mathbf{E}'_{gk}(\mathbf{g}_a) \right], \tag{6.3.1}$$

$$\mathbf{B}_g = -\frac{N_0}{2} \left\{ \sum_{k=1}^{N} \left[\mathbf{Z}''_{gk}(\mathbf{g}_a) - \frac{1}{2} \mathbf{E}''_{gk}(\mathbf{g}_a) \right] \right\}^{-1}, \tag{6.3.2}$$ /156

in which the signal integration in defining the derivatives is conducted within the limits of each of the coherence intervals, and summation is at all intervals N. The derivatives are taken according to component vectors g at the point of their a priori value.

Subsequently, matrix W_g is necessary to define the expression

$$\mathbf{W}_g = \frac{2}{N_0} \sum_{k=1}^{N} \left[\mathbf{Z}'_{gk}(\mathbf{g}_a) - \frac{1}{2} \mathbf{E}'_{gk}(\mathbf{g}_a) \right], \tag{6.3.3}$$

and also the matrix which is inverse to correlation matrix B_g.

It will be designated by U_g.

If an evaluation of accuracy is carried out at first on the basis of the time-space filtration of the signal received and corrections Δg to initial parameters g (the initial conditions of motion in a rectangular system of reference) are defined by means of solving equations (6.3.1) and (6.3.2), and then the problem is posed of obtaining evaluations of final parameters q; the corrections vector Δq can be calculated on the basis of the transformation formulas shown above:

$$\Delta q = P^{-1} \Delta g, \tag{6.3.4}$$

where $P = \delta g/\delta q$ is the transition matrix connecting the differentials of the position vector components given in the initial and terminal parameter areas.

The error correlation matrix B_q for defining parameters q can be calculated by transforming correlation matrix B_g [15, 19]:

$$\mathbf{B}_q = \mathbf{P}^{-1}\mathbf{B}_g(\mathbf{P}^{-1})^T. \tag{6.3.5}$$

In directly making terminal parameters q more precise in the received signal field, the correction vector Δq and the correlation matrix are defined by the expressions

$$\Delta \mathbf{q} = \frac{2}{N_0}\mathbf{B}_q\sum_{k=1}^{N}\left[\mathbf{Z}'_{qk}(\mathbf{q}_a) - \frac{1}{2}\mathbf{E}'_{qk}(\mathbf{q}_a)\right], \tag{6.3.6}$$

$$\mathbf{B}_q = -\frac{N_0}{2}\left\{\sum_{k=1}^{N}\left[\mathbf{Z}''_{qk}(\mathbf{q}_a) - \frac{1}{2}\mathbf{E}''_{qk}(\mathbf{q}_a)\right]\right\}^{-1}. \tag{6.3.7}$$ /157

Taking into account that both systems of parameters g and q describe the space-time position of a material object moving according to the same principle, each of the terms of the sum of first and second derivatives according to the components of vector q can be represented by the relations

$$Z'_q(q_a) - \frac{1}{2} E'_q(q_a) = P^T [Z'_g(g_a) - \frac{1}{2} E_g(g_a)], \quad (6.3.8)$$

$$Z''_q(q_a) - \frac{1}{2} E''_q(q_a) = P^T [Z''_g(g_a) - \frac{1}{2} E''_g(g_a)] P. \quad (6.3.9)$$

The value of the elements of transition matrix P is not a function of the coherence interval. The terms in brackets, representing derivatives according to the elements of vector g, are functions of its components, given by its a priori values. In substituting expressions (6.3.8) and (6.3.9) in formulas (6.3.6) and (6.3.7), the latter can be reduced to relations (6.3.4) and (6.3.5), respectively. Therefore, it is possible to conclude that, regardless of whether the terminal parameters of motion q are evaluated directly or are obtained by means of reducing the results of evaluating initial parameters g by using transition matrix P, corrections Δq to the precise parameters and their correlation matrix B_q are always identical.

The error vector correlation matrix can be used as a property in evaluating the exact properties for defining the parameters of motion. However, in practice, the use of such an exact property [19] as the correlation matrix determinant whose value, correct to constant factors, is defined by the volume of the multidimensional error ellipsoid in evaluating the chosen set of parameters, is more convenient. The connection between the volume of the error ellipsoid and the determinant can be represented by the following function [15]:

$$V_e = \pi^{m/2} \sqrt{\det B_g} \Big/ \Gamma\left(\frac{m}{2}+1\right), \quad (6.3.10)$$

where m is the dimensionality of the ellipsoid, defined by the dimensionality of the multidimensional space of parameters g; $\Gamma(m/2+1)$ is the gamma function.

In this connection, if the volume of a multidimensional dispersion ellipsoid in the space of the selected parameters is used for evaluating the exact property for defining the parameters of motion, for a quantitative evaluation of the precision properties and charateristics for defining different systems of parameters of motion, it is sufficient to know the /158 determinant of the error correlation matrix B_g for defining the components of the initial system of parameters g and the determinant of the transition matrix P, since knowledge of the latter offers the possibility of computing the determinant of error correlation matrix B_q in evaluating terminal parameters q. With the same dimensionality of multidimensional spaces of parameters g and q, the relation is correct

$$\det B_q = \det B_g/(\det P)^2. \qquad (6.3.11)$$

Thus, in m-dimensional spaces with the same metrics in the range of the parameters' definition, where the value of the determinant of the ACF signal second derivative matrix is high, the volume of the dispersion ellipsoid is small, which attests to the high accuracy in defining the parameters of motion. The reverse is also true: In an area of space where the determinant of matrix U_g is small, the accuracy of the determinant is small, and the definition accuracy is not high. Therefore, in establishing ranges within the limits of which $U_g \to 0$, we can judge the distribution of measuring complex areas in using the selected set of parameters.

We should add that, since at present the solution of systems of nonlinear equations with respect to the defined parameters is done by a method of successive approximations, the solution process will contain a smaller number of iteration cycles the larger the determinant of the second derivative matrix is. With a decrease in the determinant, the velocity of convergence becomes less; in this connection, for a convergent solution, more precise a priori data for forming the reference signal are required. A solution in the area of the parameter definition in which $\det U_g \to 0$ becomes especially difficult. The magnitude of the determinant of matrix U_g is a function of the distribution of elementary receiving antennas in space, the statistical characteristics of measurement errors, the geometric conditions of observation of the SV, and the choice of a system of initial parameters used for describing its space-time position. In defining terminal parameters q, the nature of the change in the determinant of ACF signal second derivative matrix U_q, being a function of the values of defined components q, can be studied by means of the transition matrix /159

$$\det U_q = \det U_g(\det P)^2. \qquad (6.3.12)$$

A characteristic feature of transition matrix P, in the case of its nonorthogonality, is the functional dependence of its elements and determinant on the components of parameters q. Investigating this feature allows us to study the influence of the range of definition of evaluated parameters of motion q on the accuracy of their definition. It is evident that the discontinuity of matrix P or its poor conditionality results in the discontinuity or poor conditionality of matrix U_q, also. The last leads not only to an increase in the volume of the multidimensional dispersion ellipsoid, but also to an increase in the error dispersions in defining the separate components of position vector q, whose values can be calculated by means of the relation

$$\sigma_{q_j} = \frac{\det(P_j^T U_g P_j)}{\det U_g (\det P)^2}, \quad (6.3.13)$$

where P_j is a matrix of dimensions m x (m-1), derived from transition matrix P by means of deleting the j^{th} column.

Let us note that the relations obtained which connect the correlation matrices of errors in defining parameters g and q and their determinant are correct not only where the parameters of motion are defined as a result of the space-time filtration of a signal with regularly changing amplitude and fluctuating initial phase, but also where any other signal model is used in the measuring complex (see §3.1). These expressions are correct with the presence of matrix B_{ga} of errors in a priori data, which must undergo a transformation in conformity with expression (6.3.5) in the transition from initial to terminal parameters. Moreover, these relations also take place in using an automatic mode for measuring parameters with a priori data in systems of space-time filtration, when the evaluation of the parameter vector at a defined moment of time t_0 is done in proportion to the signal integration, and a priori data continuously changes with the value of the corrections obtained, approaching its real value.

The relations derived will also remain valid for the case where a complex of several electronic systems are used, distinguished either by the parameters of the signal used, or by the spatial configurations of the receiving antennas, etc. Actually, due to the linearity of the equations examined, which are used in defining the corrections to the precise parameters, and that the corresponding elements of different matrices W and U have the same physical dimensions, the corrections vector Δq and the integrated correlation matrix of errors in defining parameters q can be represented by the expressions /160

$$\Delta q = B_{qS} \sum_{n=1}^{S} W_{q_n}, \quad (6.3.14)$$

$$\mathbf{W}_{qS} = \left(\sum_{n=1}^{S} \mathbf{U}_{q_n} \right)^{-1}, \tag{6.3.15}$$

where

$$\mathbf{W}_{q_n} = \frac{2}{N_0} \sum_{k=1}^{N_n} \left[\mathbf{Z}'_{qk}(\mathbf{q}_a) - \frac{1}{2} \mathbf{E}'_{qk}(\mathbf{q}_a) \right]$$

is the matrix of ACF signal first partial derivatives of the n^{th} measuring system according to parameters q:

$$\mathbf{U}_{q_n} = -\frac{2}{N_0} \sum_{k=1}^{N_n} \left[\mathbf{Z}''_{qk}(\mathbf{q}_a) - \frac{1}{2} \mathbf{E}''_{qk}(\mathbf{q}_a) \right].$$

is the matrix of ACF signal second partial derivatives of the n^{th} system. Applying transformations (6.3.8) and (6.3.9) to matrices W_{qn} and U_{qn}, and inserting the relations derived in expressions (6.3.14) and (6.3.15), we obtain

$$\Delta\mathbf{q} = \left(\sum_{n=1}^{S} \mathbf{P}^{\tau} \mathbf{U}_{g_n} \mathbf{P} \right)^{-1} \sum_{n=1}^{S} \mathbf{P}^{\tau} \mathbf{W}_{g_n},$$

$$\mathbf{B}_{qS} = \left(\sum_{n=1}^{S} \mathbf{P}^{\tau} \mathbf{U}_{g_n} \mathbf{P} \right)^{-1},$$

which are equivalent to relations

$$\Delta q = P^{-1} \Delta g, \tag{6.3.16}$$

$$B_{qS} = P^{-1} B_{gS} (P^{-1})^{T}. \tag{6.3.17}$$

In formula (6.3.17), matrix B_{gS} is an integrated matrix of errors in defining parameters g in the complex. /161

Thus, for a complex of different electronic meters, the precise properties for defining parameters of motion in different systems of reference can also be represented in the form of a set of properties for definitions with the use of the initial system of parameters g and the properties of transition matrix P to terminal parameters of motion q. The nature of the transformation of errors in evaluating parameters in a complex of different measurement systems will remain the same as in using any one of them. However, the accuracy of the evaluations obtained is significantly improved.

In defining, in addition to the parameters of motion, the systematic error in producing the ACF signal, as a function, in particular, of the constant mismatch of frequencies of the received and reference signals, the secondarily evaluated parameter can be interpreted as an additional coordinate in the systems of reference examined. The dimensionality of the enlarged systems of reference becomes a unit larger, since the axis of the additionally evaluated parameter (of frequency correction) becomes their axis. It is characteristic that in all systems of reference examined, this additional parameter will be the same. Therefore, the peculiarity of the transition matrix between the enlarged systems of reference is that the relative character of the error transformations in evaluating these sets of parameters will be preserved during the transition from one system of reference to another, the same as with their combined definition.

From expression (6.3.11), we can see that for a complete quantitative characterization of the accuracy of defining the parameters of motion in the newly selected systems of reference, we must know the determinant of the error correlation matrix for evaluating the parameters of initial system g and transition matrix P. In this connection, if the more precise properties for defining initial parameters g are investigated, then for a comparative analysis of the precision characteristics for defining other systems of parameters q, it is sufficient in some cases to study the properties of the matrices of transition from initial to terminal parameters, by means of which the results /162
of orbital, navigational or geodetic definitions will be represented. Of the entire set of newly applicable systems of parameters q, whose components have identical dimensions, the best one is the one whose transition matrix determinant will be greatest in the entire range of definition of the parameters or its individual, practically used range.

5.4. Method of Analyzing Transition Matrices

The purpose of investigation transition matrix P between the differential components of the position vector, given in the area of initial conditions g of a rectangular system of coordinates OXYZ (Fig. 6.1) and the selected terminal parameters g, is the definition of those areas of definition of the individual elements in a finite system of parameters where the transition matrix becomes singular. These areas will be called "zones of reduced accuracy." Moreover, it is necessary to define those areas of parameter definition where the elements of the transition matrix do not exist or are close to these values, and there is practically no possibility of solving the problem of defining the selected set of parameters.

For finding the zones of reduced accuracy in defining different sets of parameters, it is advisable to represent position vector g in the form of the product of some matrix factors whose elements are functions of one or several parameters. Such a representation of position vector g allows us to write transition matrix P and its elements rather compactly, and is also an analytical means of investigating the properties of the transition matrix.

We should note that the elements of the transition matrix which represent scalar functions can also be expressed by means of the product of the matrix factors. The proposed method of investigation does not require representing the elements of the transition matrix in an expanded form or varying the values of the orbit parameters in analyzing each of the matrix elements separately. This method assumes that transition matrix P is in the form of a matrix row of vectors, and that these vectors are investigated up to their degeneration to zero-points, and that the proportionality between them is defined.

Moreover, the representation of the transition matrix elements in the form of the product of the matrix factors offers the possibility of deriving the determinant of this matrix by an analytic method. In this connection, the process of calculating the determinant and analyzing the matrix is significantly simplified, since those elements which yield similar terms during mathematical transformations are excluded from the calculated relations.

We will demonstrate the method of analyzing transition matrices by analyzing the matrices of transition between differential components of a position vector, given in the area of initial conditions of a rectangular coordinate system OXYZ, and the Keplerian elements of orbit:

$$q^T = ||i \ \omega \ \Omega \ e \ a \ M_0||.$$

1. The fundamental form of representing transition matrices /163

For investigating the properties of transition matrices which define the transformation of errors in navigational or geodetic definitions in converting from one system of parameters to another, and also for definition errors as a function of the area of definition of the evaluated parameters, it is sufficient to study the properties of these matrices only in the form of (6.2.23). Therefore, such a notational form is fundamental. Actually, the expression which defines the relationship of the ACF signal second derivative matrices in defining the components of initial g and terminal systems of parameters q, can in the general case, be represented by the following function:

$$\mathbf{U_q} = \left\| \begin{matrix} \mathbf{P}^{\mathrm{T}}_{\cdot 1}\mathbf{U_g}\mathbf{P}_{\cdot 1} & \mathbf{P}^{\mathrm{T}}_{\cdot 1}\mathbf{U_g}\mathbf{P}_{\cdot 2} & \cdots & \mathbf{P}^{\mathrm{T}}_{\cdot 1}\mathbf{U_g}\mathbf{P}_{\cdot m} \\ \mathbf{P}^{\mathrm{T}}_{\cdot 2}\mathbf{U_g}\mathbf{P}_{\cdot 1} & \mathbf{P}^{\mathrm{T}}_{\cdot 2}\mathbf{U_g}\mathbf{P}_{\cdot 2} & \cdots & \mathbf{P}^{\mathrm{T}}_{\cdot 2}\mathbf{U_g}\mathbf{P}_{\cdot m} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{P}^{\mathrm{T}}_{\cdot m}\mathbf{U_g}\mathbf{P}_{\cdot 1} & \mathbf{P}^{\mathrm{T}}_{\cdot m}\mathbf{U_g}\mathbf{P}_{\cdot 2} & \cdots & \mathbf{P}^{\mathrm{T}}_{\cdot m}\mathbf{U_g}\mathbf{P}_{\cdot m} \end{matrix} \right\|, \tag{6.4.1}$$

which shows that with non-null matrix U_g, matrix U_q will be singular (det U_q=0) when one or several columns $P_{.j}$ are null, or there is proportionality between them.

Thus, for example, if the j^{th} column of matrix P is a null vector, then for all n and k (n, k=1, 2, ..., m), we will have the identity

$$\boxed{\mathbf{P}^{\mathrm{T}}_{\cdot j}\mathbf{U_g}\mathbf{P}_{\cdot n} \equiv \mathbf{P}^{\mathrm{T}}_{\cdot k}\mathbf{U_g}\mathbf{P}_{\cdot j} \equiv 0,} \tag{6.4.2}$$

i.e., the j^{th} row and the j^{th} column of matrix U_q will degenerate to zero. If, between the j^{th} and i^{th} columns of matrix P, proportionality (let $P_{.j} = P_{.i}$) is observed, then for all elements of matrix U_q which contain the given columns, the relation will be true

$$\boxed{\mathbf{P}^{\mathrm{T}}_{\cdot k}\mathbf{U_g}\mathbf{P}_{\cdot j}/\mathbf{P}^{\mathrm{T}}_{\cdot k}\mathbf{U_g}\mathbf{P}_{\cdot i} \equiv \mathbf{P}^{\mathrm{T}}_{\cdot j}\mathbf{U_g}\mathbf{P}_{\cdot n}/\mathbf{P}^{\mathrm{T}}_{\cdot i}\mathbf{U_g}\mathbf{P}_{\cdot n} \equiv \alpha.} \tag{6.4.3}$$

This corroborates the fact that between the j^{th} and i^{th} columns (rows) of matrix U_q there is proportionality, i.e., matrix U_q is singular. Therefore, in investigating the effect of a transition matrix on the singularity of defining parameters q, it is sufficient to study only the columns of these matrices.

2. Criteria for analyzing transition matrices

In order to find the area of reduced accuracy in defining Keplerian parameters of orbit, due to the characteristic features of transition matrices which directly reflect the properties of the indicated parameters' space, it is necessary to define the areas of definition of Keplerian parameters of orbit for which the determinant of matrix P has a minimum value or is equal to zero. This will be the area of space of the Keplerian elements in which matrix P is close to singular or is singular. /164 First of all, we will attempt to find those areas of the definition of Keplerian elements of orbit for which the determinant of the matrix approaches zero. In this connection, it will be assumed that the determinant of the matrix is equal to zero only where the columns of matrix P degenerate to a null vector or proportionality (linear function) is observed between them.

We will study a transition matrix written in the form of (6.2.23), i.e., we will examine columns $P_{.j}$ of the transition matrix in their degeneration to null vectors and in the proportionality between them.

From the preceding material, we can see that none of the matrices which comprise the factors in matrix column $P_{.f}$ degenerate to null matrices, nor with real values of the Keplerian parameters of orbit and instantaneous argument E. Therefore, elements $P_{.j}$ of the transition matrix can take zero values only in cases where the linear combination of elements of the matrices, as a function of argument E (of the elements of matrix g_1 and derivatives of its components according to parameters a, e, M_0), and the elements of other matrix factors (the elements of matrices G, H, S, and their derivatives with respect to angle (Ω, i, ω) are equal to zero.

As the criterion for the degeneration of any of the vectors $P_{.f}$ to a null vector, we can take the condition of equality to zero of the modulus of vector P, or its square. Actually, $P_{.j}$ can be a null vector only when there is the equality

$$P^T_{.j}P_{.j} = 0. \qquad (6.4.4)$$

We can accept the condition of equality to zero of the modulus of the difference of two vectors, one of which is the vector in question, for example, $P_{.j}$, and the second is the product of some real scalar magnitude α in the second vector $P_{.i}$, as the condition of the linear function (proportionality) of two vectors $P_{.f}$ and $P_{.i}$. If the product is formed by

$$(\mathbf{P}_{.j}-\alpha\mathbf{P}_{.i})^{\mathrm{T}}(\mathbf{P}_{.j}-\alpha\mathbf{P}_{.i})=\beta, \tag{6.4.5}$$

then it is possible to show that this expression will equal zero only when there is proportionality between the components of vectors $P_{.j}$ and $P_{.i}$, and α is the proportionality coefficient. Consequently, the problem of studying the proportionality of two vectors is the problem of finding a real value of coefficient α, not equal to zero, by which, considering the area of definition of parameters q, the equality will be obtained

$$(\mathbf{P}_{.j}-\alpha\mathbf{P}_{.i})^{\mathrm{T}}(\mathbf{P}_{.j}-\alpha\mathbf{P}_{.i})=0. \tag{6.4.6}$$

If this equality is derived with $\alpha=0$, then condition (6.4.4) is factually fulfilled, i.e., $P_{.j}$ is the null vector.

It is possible to show that if $P_{.j}$ and $P_{.i}$ are orthogonal, then expression (6.4.5) leads to the relation

$$\mathbf{P}^{\mathrm{T}}_{.j}\mathbf{P}_{.j}+\alpha^{2}\mathbf{P}^{\mathrm{T}}_{.i}\mathbf{P}_{.i}=\beta \tag{6.4.6}$$

and can be equal to zero in the range of definition of parameters q, in which both vectors are null, since orthogonal vectors are linearly independent. /165

We must note that condition (6.4.6) is satisfied when the Bunjakowski-Cauchy inequality, used in [4] for describing the general properties of navigational methods resulting from the features of fundamental matrices with averaged elements, is converted to an equality.

On the basis of the investigations shown in defining and analyzing the squares of the moduli of the vectors in matrix P (6.2.23), we can come to the following conclusions.

The square of the modulus of the first vector $P^T_{.1}P_1$ for elliptical motion is not equal to zero, whatever the values of the Keplerian elements are. However, when time t_0 corresponds to the moment of the SV's passage over the equator, derivatives of the coordinate components of position vector g with respect to the angle of deviation of the orbit are equal to zero. Moreover, the square of the modulus of velocity components for this vector in parabolic orbits is equal to infinity; for hyperbolic orbits, its value becomes negative. This indicates that for describing parabolic and hyperbolic orbits, it is necessary to use another system of parameters. In particular, for parabolic orbits, it is sufficient to retain only five parameters, since the sixth can be determined by the limitations superimposed on the existence of a parabolic orbit [28]. For hyperbolic orbits, the semimajor axis loses the value which it had in elliptical motion.

For elliptical orbits, the square of the modulus of the third vector $P^T_3P_{.3}$ does not have zero values. However, it shows that derivatives of the coordinates components of position vector g along the parameter of angular distance of the ascending node with i, $\omega = 90°$ and $E = 0$ (moment of time t_0 corresponds to the SV's passage of the vertex point in a polar orbit) are equal to zero. With these values for the deviation or the orbit and angular distance of the perigees for cases where moment t_0 corresponds to the SV's passing over the equator, the derivatives of the velocity components along the parameter of the angular interval of the ascending node are equal to zero.

In defining the square of the modulus of the second, fourth, and subsequent columns of matrix P, we come to the conclusion that these columns do not degenerate to zero, since the squares of their moduli are defined by the sum of the squares of the last factor's elements which, whatever the values of argument E and the other intraplanar parameters, does not degenerate to a null vector.

In investigating matrix P's proportionality between columns, data were obtained which show that between the first and second, fourth, fifth and sixth vectors (columns) of expression (6.2.23), proportionality is not observed. These vectors are mutually orthogonal, and since none of them are non-zero, then condition (6.4.6) is not satisfied.

We should emphasize that in investigating a transition matrice with proportionality between columns, special attention should be turned to the vectors which contain derivatives of the position vector components with respect to parameters of identical dimensionality (in particular, with respect to angular elements), and to those ranges of parameter definition in which, with respect to the physical expression, one element of orbit or another loses sense. Thus, investigations of expression (6.4.5) for the second and sixth columns of the matrix showed that it becomes equal to zero with α=-1 for e=0. The equality to zero of the expression referred to indicates that in this case the transition matrix becomes singular, and it is not possible to define all six Keplerian parameters. /166 This is clear, since in the given case the ellipse degenerates into a circle, and for a circle, the parameter of the angular interval of the perigee ω loses meaning. In this connection, for defining the SV's initial position in orbit, corresponding to moment t_0, one angular parameter is sufficient. For the coordinate components of these vectors, proportionality with any value of eccentricity is observed when time t_0 corresponds to the moment of the SV's passing the point of the perigee or the apogee.

Expression (6.4.5) for the second and third columns of transition matrix P in the condition where the longitude of the ascending node loses its physical sense (i=0) satisfies requirement (6.4.6). This indicates that proportionality is observed between the second and third columns of the matrix (derivatives of position vector g with respect to scalar values Ω and ω). Therefore, the Keplerian elements of orbit with small values for the angles of deviation are not very effective. For describing these orbits, we must use another system of parameters. In particular, in reference [3] it is recommended that the cosine of this angle be used instead of the angle of deviation; however, the effectiveness of such a substitution requires further investigation.

For almost circular orbits with small angles of deviation along with proportionality between the second and third columns, proportionality is also observed between the third and sixth columns of the matrix.

For defining the proportionality between columns $P_{\cdot j}$ and $P_{\cdot i}$, other criteria can be used. In particular, it is possible to introduce "angle" θ between vectors $P_{\cdot j}$ and $P_{\cdot i}$, by analogy with the scalar product of vectors in three-dimensional euclidean space, having determined it from the relation

$$\cos^2\theta = (\mathbf{P}^{\mathrm{T}}_{.j}\mathbf{P}_{.i})^2/(\mathbf{P}^{\mathrm{T}}_{.j}\mathbf{P}_{.j})(\mathbf{P}^{\mathrm{T}}_{.i}\mathbf{P}_{.i}). \qquad (6.4.8)$$

The values of the parameters which satisfy the condition $\cos^2\theta=1$ define the range of their possible definition in which matrix P becomes singular because of the proportionality of its columns. Investigations with the aid of the criterion indicated confirmed the correctness of the conclusions drawn earlier and showed that the cosine of the angle between columns $P_{.2}$ and $P_{.3}$, $P_{.2}$ and $P_{.6}$, $P_{.3}$ and $P_{.6}$ is connected correspondingly with parameters i, e, i and e by a functional dependence which with i, e $\to 0$ reduces the cosine of the angle to its maximum value.

With i, e=0 in a system of Keplerian parameters as in the system of reference used for describing the SV's position vector, there is a linear dependence between some of its components. These correspondences are the angular parameters ω, Ω, and M_0. The linear dependence leads to the same response of the ACF signal as in using the parameters indicated, which causes proportionality not only between the columns of the transition matrix, but also between the columns and rows of the ACF signal second derivative matrix with respect to defined parameters q. For orbits with eccentricity or inclination close to zero, the matrix of ACF signal second derivatives, depending on the position of the measuring agents used and their complex, is badly specified, resulting in significant errors in the definitions. The conditionality of the matrix can be improved by using other elements of orbit.

Thus, these investigations show that the zones of reduced accuracy, due to the characteristic properties of the space of the Keplerian parameters of orbit (the latter are refracted /167 in the properties of transition matrix P), are observed in the ranges of their definition in which one or several elements lose physical sense. In this connection, for a unique definition of the SV's movement, it is possible to use a smaller quantity of independent generalized parameters (the case of degeneration of an ellipse into a circle or parabola) or to introduce new parameters of orbit (the case of elliptical equatorial and hyperbolic orbits) instead of the Keplerian elements of orbit.

For elliptical equatorial orbits (i=0) and for the case where elliptical orbits degenerate into circular orbits, transition matrix P becomes singular and it is not possible to define all six Keplerian parameters.

In the proximity of setting the Keplerian orbit parameters directly adjacent to the values in which transition matrix P is singular, this matrix will be close to singular, which causes significant zones of reduced accuracy in defining the parameters of motion q to appear.

5.5. Characteristics of Parameter Definition in Rectangular and Curvilinear Systems of Reference

1. Rectangular systems

As the initial system of parameters g, we will choose a system of elements which describe the initial conditions of the SV's motion in some rectangular system of reference, for example, in a system of coordinates connected with the observer. Then, selecting the coordinates and velocity of the SV in any other rectangular coordinate system q, for example, in different geocentric systems of reference, we will discover that, due to the orthogonality of the matrix transition which describes the transformation of differentials (6.2.5), the modulus of its determinant is equal to one. This means that the possibilities of making the parameters of motion more precise in an arbitrary rectangular system of reference are not functions of the range of definition of the coordinates and velocity and are identical for all rectangular coordinate systems.

2. Curvilinear systems

For the sake of convenience, we will examine the rectangular geocentric system of reference whose plane OXY coincides with the basic plane of the curvilinear systems of coordinates, and axis OX coincides with some characteristic direction with respect to which angles are read in curvilinear systems, as the initial system. In this connection, by transition matrices P, we will imply one of the matrices defined by relation (6.2.15). The transformation matrices indicated are quasidiagonal; therefore, their determinants are expressed by the determinant of direct transformation matrices of differentials W_C, W_S and W_g. The latter can easily be defined, since differential direct transformation matrices are diagonal (6.2.12).

Thus, the determinant of the transition matrix of differentials of initial conditions of motion in a spherical coordinate system is equal to /168

$$\boxed{\det P_S = (\det J_S)^2 = (\det \ddot{W}_S)^2.} \qquad (6.5.1)$$

The relation of the determinants of error correlation matrices in making the parameters of motion more precise for rectangular and spherical coordinate systems is expressed by the function

$$\det \mathbf{B}_{q_S} = \det \mathbf{B}_{g_{tr}} / r_K^8 \cos^4 \varphi, \tag{6.5.2}$$

which shows that the volume of the dispersion ellipsoid in defining the initial conditions of motion in a spherical system of reference is increased when r_K decreases and ϕ increases. In confirmation of this, the graph of the function which describes the change in the dimensions of the volume of a multidimensional dispersion ellipsoid in using latitude ϕ and relation r_K/r_{Kmin}, where r_{Kmin} is the minimally possible radius, is shown in Fig. 6.2. If the SV's motion occurs in a circular orbit, then the dimensions of the dispersion range for defining

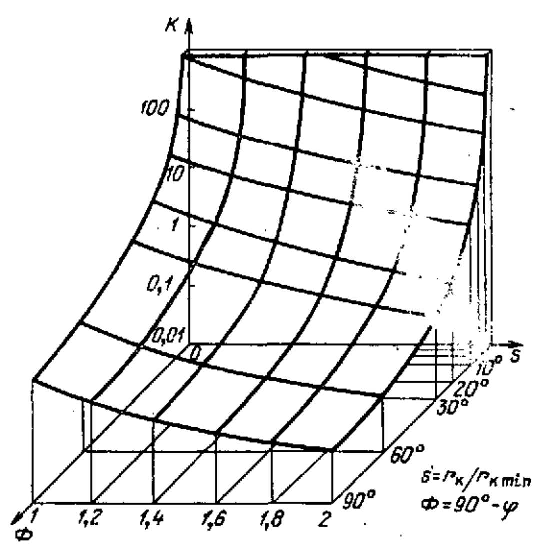

Fig. 6.2. The nature of the error ellipsoid's volume change with a change in spherical latitude and relation r_K/r_{Kmin}.

the parameters of motion in a sperhical system of coordinates will be a function of only the latitude of the SV's position.

In the figure, this function for several fixed values of radius vector r_K is indicated by the solid lines which lie in the planes parallel to coordinate plane KOΦ.

In converting to parameters of a cylindrical system of coordinates, the expression corresponding to the transformation of numerical values of the volume of multidimensional ellipsoids described by error correlation matrices for making the components of the position vector more precise, given by the initial conditions of motion in rectangular and cylindrical systems of reference, have the form /169

$$\sqrt{\det B_{q_C}} = \sqrt{\det B_{g_{tr}}} / \rho^2. \qquad (6.5.3)$$

Expression (6.5.3) shows that the dimensions of the dispersion range in evaluating the parameters of motion in a cylindrical system of coordinates are a function of only one component ρ of the SV's position in a given system of reference and are not functions of other components of the parameters. With a decrease in coordinate ρ, the dimensions of the dispersion range increase, since errors in defining the angular parameter in a cylindrical system increase with identical linear errors along the parallel. They are inversely proportional to parameter ρ. Note that with $\rho=0$, the transition matrix, correspondingly, and the matrix of ACF signal second derivatives, with a direct definition of the parameters of motion in a cylindrical coordinate system, become singular. It is true that $\rho=0$ loses sense, the SV position parameter is one like λ_C, and as a consequence, the response of the ACF signal at this coordinate disappears. Therefore, for solving the problems of defining the parameters of motion of a polar SV in a given system of coordinates over a pole or in its proximity, an increase in errors is observed.

Thus, in using cylindrical or spherical systems of reference, congruence of the reference coordinate plane of these systems with the plane of orbit is considered more preferable, as recommended in references [25, 26] for a cylindrical system of coordinates. Let us recall that simplification of algorithms for processing measurement data and a corresponding decrease in computer time [25] are also obtained. It should also be noted that in this case, since $\cos\phi=1$ and $r_K=\rho$, the cylindrical and spherical systems of coordinates are equivalent, and only the magnitude of the defined radius vector of the SV influences the definition of the parameters of motion in these systems of reference. However, in the general case, cylindrical and spherical systems of coordinates are nonequivalent. A spherical system of reference which contains two angular parameters λ_S and ϕ is more sensitive to a change in the SV's radius vector than a cylindrical system.

The determinant of the transition matrix which establishes the relation between dispersion ellipsoids in defining the initial conditions of the SV's motion in geodetic and rectangular geocentric systems of coordinates is calculated by means of the relation

$$\det \mathbf{P}_g = (\det \mathbf{W}_g)^2 = (N_1 + H)^2 (N + H)^2 \cos^2 B_g \qquad (6.5.4)$$

which can be reduced to the following expression:

$$\det \mathbf{P}_g = \left[s^2 + \frac{2 - e_3^2 (1 + \sin^2 B_g)}{(1 - e_3^2 \sin^2 B_g)^{3/2}} s + \frac{1 - e_3^2}{(1 - e_3^2 \sin^2 B_g)^2} \right]^2 \times a_3^4 \cos^2 B_g, \qquad (6.5.5)$$

where $s = H/a_3$.

Thus in making the parameters of motion more precise in a geodetic system of reference, the numerical value of the volume of the multidimensional dispersion ellipsoid is a function both of the coordinates at moment t_0 of the precise parameters, and of the elements of the reference ellipsoid with respect to /170 the plane of which the definition was made. In evaluating the parameters of motion, the properties of the definitions are a function of the geodetic latitude B_g and altitude H of the SV with respect to the surface of the reference ellipsoid. With the dimensions of the area of dispersion given in a rectangular system of coordinates, the number values of the error ellipsoid's volume in a geodetic system of reference increase with a decrease in altitude H and an increase of latitude B_g. The function below is shown in Fig. 6.3

$$K = \frac{(1 - e_3^2)^2}{\left[s^2 + \frac{2 - e_3^2(1 + \sin^2 B_g)}{(1 - e_3^2 \sin^2 B_g)^{3/2}} s + \frac{1 - e_3^2}{(1 - e_3^2 \sin^2 B_g)^2} \right]^2 \cos^2 B_g}, \qquad (6.5.6)$$

which describes the change in the dimensions of the dispersion area as a function of the geodetic latitude B_g and the relations of the altitude of the SV's position at moment t_0 over the plane of the reference ellipsoid with respect to its semimajor axis. For actually existing orbits, the zones of reduced accuracy in defining the initial conditions of motion in a geodetic coordinate system coincide with the range of their definition, for which $B_g \to 90°$. More marked deterioration in accuracy is observed for polar orbits when the moment of time t_0 coincides with the moment of the SV's passage over the pole or over its proximity in which geodetic latitude B_g exceeds 80°. With $B_g = 90°$, transition matrix P_g becomes singular. Therefore, the matrix of ACF signal second derivatives, with direct definition of the /171 parameters of motion in a geodetic system of reference, will be singular. Moreover, in some ranges of definition of parameters q_g, which adjoin the point of multidimensional space with a latitude equal to 90°, the value of the determinant of the ACF second derivative matrix will be small, which, as a rule, results in the instability of the inverse correlation matrix and a marked increase in the error dispersions for defining the selected set of parameters. The instability of the correlation matrix, as we know, is a function of the fact that for small values of the ACF signal second derivative matrix determinant, small changes in its elements cause a change in the elements of the inverse matrix within significant limits. The appearance of zones of reduced accuracy for regions of element definition in the chosen system of parameters, in which the determinant of the transition matrix and the matrix of ACF signal second derivatives approach zero is explained by this.

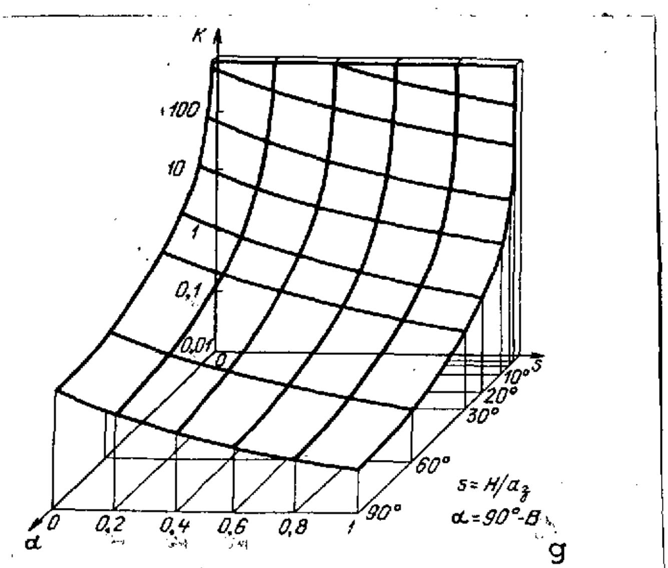

Fig. 6.3. The nature of changes in the error ellipsoid volume with changes in the geodetic latitude and relation H/a_z

6.6. Properties of Keplerian Elements of Orbit

As above, we will examine a geocentric rectangular equatorial system of coordinates as the initial system, and the system of parameters $q^T = ||i\ \omega\ \Omega\ a\ e\ M_0||$ as the system of Keplerian elements of orbit.

An investigation of the properties of electronic methods for defining Keplerian parameters of orbit will be carried out by studying the properties of a transition matrix which describes the transformation of errors in definitions in converting from initial conditions of motion in a rectangular system of reference to the Keplerian elements mentioned. In this connection, since the volume of a multidimensional dispersion ellipsoid is used for describing the accuracy of defining the parameters, whose numerical value is equal to the determinant of a correlation matrix (which includes a transition matrix) with accuracy to constants, for studying the indicated properties, it is sufficient to calculate the determinant of this matrix and investigate its value as a function of the region of definition of the individual Keplerian elements of orbit.

For finding the determinant of the matrix indicated, we will use relation (6.2.39). In this connection, since matrices G, H and S are orthogonal, the determinant of transition matrix P is identically equal to the determinant of the direct differential transformation matrix W_K. Therefore, decomposing this determinant by the elements of the first column, with each determinant obtained being of the fifth order with respect to the elements in the row which contain only one non-zero element, after the corresponding mathematical transformations, we obtain a fairly compact expression for calculating the determinant of the transition matrix. Having represented this expression in the form of a function of coordinate and velocity components of vector g_1 and their derivatives with respect to intraorbital Keplerian elements, we derive

$$\det \boldsymbol{P} = (x_1 \dot{y}_1 - \dot{y}_1 x_1)\left[\left(x_1 \frac{\partial x_1}{\partial a} + y_1 \frac{\partial y_1}{\partial a}\right)\left(\frac{\partial \dot{x}_1}{\partial e} \cdot \frac{\partial \dot{y}_1}{\partial M_0} - \right.\right.$$

$$\left. - \frac{\partial \dot{y}_1}{\partial e} \cdot \frac{\partial \dot{x}_1}{\partial M_0}\right) - \left(x_1 \frac{\partial x_1}{\partial e} + y_1 \frac{\partial y_1}{\partial e}\right)\left(\frac{\partial \dot{x}_1}{\partial a} \cdot \frac{\partial \dot{y}_1}{\partial M_0} - \right.$$

$$
\begin{aligned}
& - \frac{\partial \dot{y}_1}{\partial a} \cdot \frac{\partial \dot{x}_1}{\partial M_0}\Big) + \Big(x_1 \frac{\partial x_1}{\partial M_0} + y_1 \frac{\partial y_1}{\partial M_0}\Big)\Big(\frac{\partial \dot{x}_1}{\partial a} \cdot \frac{\partial \dot{y}_1}{\partial e} - \\
& - \frac{\partial \dot{y}_1}{\partial a} \cdot \frac{\partial \dot{x}_1}{\partial e}\Big) + \Big(\dot{x}_1 \frac{\partial \dot{x}_1}{\partial a} + \dot{y}_1 \frac{\partial \dot{y}_1}{\partial a}\Big)\Big(\frac{\partial x_1}{\partial e} \cdot \frac{\partial y_1}{\partial M_0} - \\
& - \frac{\partial y_1}{\partial e} \cdot \frac{\partial x_1}{\partial M_0}\Big) - \Big(\dot{x}_1 \frac{\partial \dot{x}_1}{\partial e} + \dot{y}_1 \frac{\partial \dot{y}_1}{\partial e}\Big)\Big(\frac{\partial x_1}{\partial a} \cdot \frac{\partial y_1}{\partial M_0} - \\
& - \frac{\partial y_1}{\partial a} \cdot \frac{\partial x_1}{\partial M_0}\Big) - \Big(\dot{x}_1 \frac{\partial \dot{x}_1}{\partial M_0} + \dot{y}_1 \frac{\partial \dot{y}_1}{\partial M_0}\Big)\Big(\frac{\partial x_1}{\partial a} \cdot \frac{\partial y_1}{\partial e} - \\
& \qquad - \frac{\partial y_1}{\partial a} \cdot \frac{\partial x_1}{\partial e}\Big)\Big] \sin i.
\end{aligned}
\tag{6.6.1}
$$

/172

Expression (6.6.1) shows that it is sufficient to know the projection of position vector g_1 on the axis of the orbital system of coordinates $OX_1Y_1Z_1$ and their derivatives with respect to the intraplanar Keplerian parameters for finding the determinant of transition matrix P. Moreover, the determinant of transition matrix P and, correspondingly, the feasibility and accuracy of the definitions when Keplerian parameters are used, is not a function of the longitude of the ascending node and the argument of the perigee, and is completely defined by the angle of deviation of the orbit and the intraorbital elements. The value of the determinant of transition matrix P with the orbit nearing an equatorial orbit is decreased, and for an equatorial orbit, is equal to zero; this is identical to the increase in the volume of the multi-dimensional error ellipsoid for defining the Keplerian parameters.

We will transform expression (6.6.1) by substituting the corresponding components of vector g_1 and their derivatives for the intraorbital Keplerian elements a, e and M_0, taken from §6.2. As a result of this substitution and the execution of a number of transformations, we obtain the expression for defining the matrix of transition to Keplerian parameters

$$
\det \mathbf{P} = -\frac{e\mu\sqrt{\mu a}}{2} \sin i. \tag{6.6.2}
$$

As we can see, definition of the matrix of transition to Keplerian parameters is a function of only three elements: α, e and i. Moreover, it is a function of the gravitation constant of central body μ around which the SV rotates.

The essential moment is when the eccentric or true anomaly is taken as the independent variable, and the determinant of transition matrix P is not a function of the indicated instantaneous variables corresponding to moment t_0. This attests to the fact that the accoracy in defining Keplerian parameters in the sense of the volume of a multidimensional error ellipsoid is not a function of which moment of time the initial conditions are defined in a rectangular system. The latter, naturally, is true only in the case where the determinant of the correlation matrix of errors in defining the initial conditions is not a function of moment t_0.

From expression (6.6.2), it follows that transition matrix P between the differentials of the SV's position vector components, given by the initial conditions of a rectangular system and by the Keplerian elements, is related to a class of nonorthogonal matrices. In this connection, since the determinant of transition matrix P is a function of some Keplerian elements, we can assume that a system of Keplerian parameters is a multidimensional, nonorthogonal, special oblique-angled system of reference. The distinctive feature of the given oblique-angled system of reference is that the relative position of its vectors is a function of the magnitude of eccentricity e and deviation of orbit i. /173

The functional dependence of the relative position of the basis vectors on the indicated Keplerian elements results in the fact that, with a change in the values of eccentricity e and angle of deviation i, not only the form of the multidimensional error ellipsoid for definign parameters q is changed, but also its volume. Thus, with a decrease in the eccentricity or deviation, the volume of the ellipsoid increases, and in the limiting case where $e \to 0$ or $i \to 0$, its value approaches infinity. This is a result of the degeneration of the hexaparametric system of Keplerian parameters, due to which proportionality is observed between columns which are a product of vector g with respect to the angular interval of the perigee and the mean anomaly if $e = 0$. With $i=0$, there is proportionality between the derivatives of the components of vector g with respect to the parameters of longitude of the ascending node Ω and the angular interval of the perigee ω.

Thus, the zones of reduced accuracy in defining the Keplerian parameters directly border upon the areas of their definition, in which one or several elements lose physical sense. The loss of physical sense by the individual parameters is due to the degeneration of the hexaparametric system of Keplerian elements into a system with a smaller number of parameters. Thus, for example, for describing the angular position of the SV with movement in a circular equatorial orbit, instead of three elements Ω, ω and M_0, only one angular parameter equal to the sum of the latter should be introduced. Let us note that the use of systems with a smaller number of parameters does not allow us to describe the space-time position of the SV precisely enough with its movement in almost circular and almost equatorial orbits. For characterizing similar orbits, as in the general case, knowledge of the numerical values of the six independent constants is necessary. However, it is expedient to use the system of Keplerian parameters for defining the elements of the orbits mentioned above. For these orbits, other systems of parameters should be used, among which are systems which contain elements representing a linear combination of the angular interval of the ascending node, the perigee and the mean anomaly.

Expression (6.6.2) offers the possibility not only of defining the position of the zones of reduced accuracy resulting from the properties of the space of the Keplerian parameters of orbit, but also quantitatively evaluating the deterioration in the accuracy of definitions with respect to the increase in volume of the multidimensional ellipsoid of errors in transition to these zones. Since the determinant of transition matrix P is a component part of the determinant of the correlation matrix of errors in evaluating Keplerian parameters which, with accuracy to constant factors, is numerically equal to the volume of the multidimensional dispersion ellipsoid, then by means of expressions (6.3.10) and (6.3.11), we can show that the relation

$$K = |\det \mathbf{P}|_{max} / |\det \mathbf{P}|, \quad (6.6.3)$$

where $[\det P]_{max}$ is the maximum value of the determinant of transition matrix P, can be written in the form /174

$$K = V_e / V_{e\ min}, \quad (6.6.4)$$

where V_e is the volume of the ellipsoid of errors in defining Keplerian parameters of orbit with any assignment of eccentricity and angle of deviation; $V_{e\ min}$ is the volume of the error ellipsoid with e=1 and i=90°.

It is natural that between relations (6.6.3) and (6.6.4) there is equality only where the determinant of the correlation matrix of errors in making the initial conditions of motion more precise in a rectangular system with a change in eccentricity and the angles of deviation in orbit remain constant.

If the value of the semimajor axis of Keplerian orbit remains constant, then magnitude K describes the change in the dimensions of the multidimensional area of dispersion with changes in eccentricity and angle of deviation (Fig. 6.4).

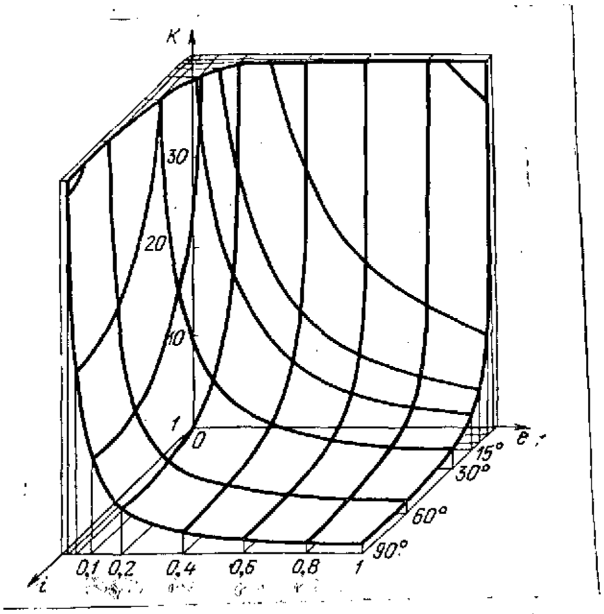

Fig. 6.4. Nature of the change in the volume of the error ellipsoid with a change in the eccentricity and angle of deviation of the orbit.

With the same values for the determinant of the correlation matrix of errors in determining the initial conditions, the dimensions of the multidimensional ellipsoid of errors in defining the Keplerian parameters attain minimum values with e=1 and i=90°. In this connection, the value of magnitude K is equal to one. With a change in eccentricity or angle of deviation to the side of their decrease, the dimensions of the error ellipsoid increases, attaining an infinitely large magnitude with e →0 or i →0. It is natural that the value of coefficient K, with the same values of eccentricity and angle of deviation, also approaches infinity.

Since with a constant value of the semimajor axis the coefficient (6.6.3) is a function of two variables K=K(e,I), then in the range of definition of parameters e and i function K(e,i) is represented by a plane. In the figure, only part of this plane, which limits the areas of definition of the eccentricity and angle of deviation within the limits of e=1-0.025; i=90°-1°.5, is shown. In this connection, the lines in the plane are the signs of the intersection of the given plane with planes parallel to coordinate, i.e., these lines show the nature of the change in coefficient K with a change in one of its Keplerian elements within the limits indicated above and with a fixed value of the other parameter. It is clearly seen that with a change in eccentricity within the limits of e=1-0.4 and /175 angle of orbital deviation within the limits of $i=90°-25°$, the volume of the multidimensional ellipsoid of errors in defining Keplerian elements is increased not more than six times. The accuracy is significantly impaired when the value of the eccentricity (angle of deviation) is lower than 0.01 (1°).

6.7. Systems of Elements Similar to Keplerian Elements. Canonical Parameters of Motion

In examining systems of elements similar to Keplerian and canonical parameters of motion, it is expedient to use an inertial geocentric rectangular coordinate system as the initial system, and as the intermediate system, a system of Keplerian elements. Therefore, for the sake of convenience, we will introduce the following product into the transition matrix between the differentials of the position vector components, given by the initial conditions of motion in a rectangular system of coordinates and the elements of the systems examined

$$P_i = PN_i.$$

in which matrix P describes the transformation of differentials in converting from an intermediate to a rectangular system of reference, and matrix N_u defines the connection between the differentials of the systems of parameters examined and the Keplerian elements of orbit which emerge as the intermediate system of reference. In this connection, for evaluating the properties of canonical parameters of motion and parameters similar to Keplerian elements, it is also necessary to analyze the properties of the determinant of matrix N_i.

1. Elements of orbits similar to Keplerian elements

As systems of parameters similar to Keplerian parameters, we will examine a modification of a system of Keplerian parameters which is described by substituting some of its elements for others which are more convenient for solving problems.

In Table 6.1, some modified systems of elements and the determinant of the matrices of secondary transition N_i and the matrix P_i are shown, and also the physically realizable values of parameters in which $\det P_i \to 0$. As we can see from the table, the use of parameters similar to Keplerian parameters leads to unequal accuracy in their definition in the entire region of assignment of these parameters. The volume of the error ellip-

Table 6.1

№№	Characteristics of the substitution	det N_i	det P_i	Values of parameters in which det $P_i \to 0$
1	a for p	$1/(1-e^2)$	$-e\mu\sqrt{a\mu}\times$ $\times \sin i/2(1-e^2)$	$e \to 0;\ i \to 0$
2	e for p	$-1/2ae$	$\mu\sqrt{\mu}\sin i/4\sqrt{a}$	$i \to 0$
3	a for T	$\sqrt{\mu}/3\pi\sqrt{a}$	$-e\mu^2 \sin i/6\pi$	$e \to 0;\ i \to 0$
4	M_0 for τ	$-\sqrt{\mu}/a\sqrt{a}$	$e\mu^2 \sin i/2a$	$e \to 0;\ i \to 0$
5	i for $\cos i$	$-1/\sin i$	$e\mu\sqrt{a\mu}/2$	$e \to 0$
6	$e,\ \omega,\ M_0$ for $\begin{cases} k = e\cos\omega;\ h = e\sin\omega; \\ M_1 = \omega + M_0 \end{cases}$	$-1/e$	$\mu\sqrt{a\mu}\sin i/2$	$i \to 0$
7	$i,\ e,\ \omega,\ M$ for $\begin{cases} b = \cos i; \\ k = e\cos\omega;\ h = e\sin\omega; \\ M_1 = \omega + M_0 \end{cases}$	$1/e\sin i$	$-\mu\sqrt{a\mu}/2$	
8	$\left.\begin{matrix} i,\ \omega, \\ \Omega,\ e, \\ M_0 \end{matrix}\right\}$ for $\begin{cases} q_1 = \sin i\cos\Omega;\ p_1 = \sin i\sin\Omega; \\ k_1 = e\cos(\omega+\Omega);\ h_1 = e\sin(\omega+\Omega); \\ M_2 = \omega + \Omega + M_0 \end{cases}$	$-1/e\sin i\cos i$	$\mu\sqrt{a\mu}/2\cos i$	$i \to 90^\circ$

№№ п/п	parameter Название и состав параметров name	association of canonical parameters and Keplerian elements	det N_i	det P_i
1	Jacobi elements $\alpha_3\, \beta_2\, \beta_3\, \alpha_1\, \alpha_2\, \beta_1$	$\alpha_3 = \sqrt{a\mu(1-e^2)}\cos i$; $\alpha_1 = -\mu/2a$; $\beta_2 = \omega$; $\alpha_2 = \sqrt{a\mu(1-e^2)}$; $\beta_3 = \Omega$; $\beta_1 = M_0\sqrt{a^3/\mu} - t_0$	$\frac{2}{e\mu\sqrt{a\mu}\sin i}$	-1
2	Delaunay parameters $H\, g\, h\, L\, G\, l$	$H = \sqrt{a\mu(1-e^2)}\cos i$; $L = \sqrt{a\mu}$; $g = \omega$; $G = \sqrt{a\mu(1-e^2)}$; $h = \Omega$; $l = M_0$	$\frac{2}{e\mu\sqrt{a\mu}\sin i}$	-1
3	1st type Poincaré parameters $\rho_2\, \omega_1\, \omega_2\, L\, \rho_1\, \lambda$	$\rho_2 = \sqrt{a\mu(1-e^2)}(1-\cos i)$; $L = \sqrt{a\mu}$ $\omega_1 = -(\omega+\Omega)$; $\rho_1 = \sqrt{a\mu}(1-\sqrt{1-e^2})$; $\omega_2 = -\Omega$; $\lambda = \omega+\Omega+M_0$	$\frac{2}{e\mu\sqrt{a\mu}\sin i}$	-1
4	2nd type Poincaré parameters $\xi_2\, \eta_1\, \xi_1\, L\, \eta_2\, \lambda$	$\xi_2 = \sqrt{2\rho_2}\cos\Omega$; $L = \sqrt{a\mu}$; $\eta_1 = \sqrt{2\rho_1}\sin(\omega+\Omega)$; $\eta_2 = \sqrt{2\rho_1}\cos(\omega+\Omega)$; $\xi_1 = \sqrt{2\rho_2}\sin\Omega$; $\lambda = \omega+\Omega+M_0$	$\frac{2}{e\mu\sqrt{a\mu}\sin i}$	-1

soid in evaluating the parameters is a function of the semimajor axis. For the majority of systems, with a decrease in the semimajor axis, the volume increases. As in the case of Keplerian parameters of orbit, the properties of elements similar to Keplerian elements do not depend on the longitude of the ascending node and the angular interval of the perigee. For actually existing orbits ($a \neq 0$), transition matrix P_i will be singular in the region of assigning defined parameters in which those separate from them (ω with $e \to 0$ and Ω with $i \to 0$) lose physical sense, and for defining the space-time position of the SV, knowledge of the smaller number of parameters is sufficient. /178

Let us note that the given tables clearly emphasize the advantage of using separate systems of elements for solving problems of making unknown parameters of motion more precise for defining the class of orbits. For example, the system of parameters q_6 for almost circular orbits, q_7 and q_8 for almost equatorial orbits do not result in the appearance of the matrix characteristics, and correspondingly offer the possibility of solving the problem of defining the chosen composition of paramaters to the end.

2. Canonical parameters

Let us examine canonical parameters of motion which are more often used in astronomy for investigating the characteristics of heavenly bodies. The canonical parameters of motion can be used successfully for describing the laws of motion of a SV.

The characteristics of several systems of canonical parameters are shown in Table 6.2. From the data shown in the table, we can see that the transition matrices which describe the relation between the differentials of initial conditions of motion in a rectangular inertial system of coordinates and the canonical parameters are orthogonal mapping matrices whose ~~whose~~ determinant, as we know, is equal to -1. Therefore, the volumes of the multidimensional ellipsoids of errors in defining the different systems of canonical parameters are identical and equal to the volume of the dispersion ellipsoid for defining the initial conditions in a rectangular system of reference. Moreover, transition from one system of canonical parameters to any other is done by means of orthogonal transition matrices.

Thus, the features noted above for defining systems of parameters of motion which have several values for the components of these systems, are due to the specific properties of multidimensional spaces of the parameters examined and are connected with losses of physical sense of the individual coordinates of cylindrical, spherical and geodetic systems, and also of individual elements of different systems of Keplerian and similar parameters of orbit, which can be eliminated by a rational transition to another system of reference. The different rectangular coordinate systems and systems of canonical parameters of motion are free of these features because of their intrinsic isometric properties.

REFERENCES

1. Agadzhanov, P.A., Dulevich, V.E. et al., Sov. Radio, "Cosmic Trajectory Measurements," Moscow (1969).

2. Amiantov, I.N. Primenenie teorii reshenii k zadacham obnaruzheniya signalov iz shumov [Application of the theory of solutions to problems of detecting signals from noise], The N. Ye. Zhukovskiy VVIA (1958).

3. Akim, E.L., Eneyev, T.M. "Defining the parameters of motion of space vehicles in terms of trajectory measurement data", Kosmicheskiye issledovaniya, 1, 1 (1963).

4. Bychkov, S.I. et al. "Cosmic electronic complexes", Sov. radio, (1967).

5. Woodward, F.M., "Probability theory and information theory with applications to radar", Sov. radio, (1955).

6. Duboshin, G.N., Nebesnaya mekhanika. Osnovnyye zadachi i metody [Stellar mechanics. Basic problems and methods], Moscow, 1961.

7. Kaula, U., "Satellite geodesy", Mir (1970).

8. King-Healey, D., "Observing Earth satellites...", Mir (1968).

9. Kolegov, G.A., "Variations in surface atmosphere density in terms of data on fluctuating periods of revolution of artificial Earth satellites", In collection: Iskusstvennyye sputniki Zemli [Artificial Earth Satellites], 4 (1960).

10. Kotel'nikov, V.A., Teoriya potentsial'noy pomekhoustoichivosti [Theory of potential noise immunity], Moscow, 1961.

11. Kotel'nikov, V.A. et al.,"Radar facility used for radiolocation of Venus in 1961", Radiotekhnika i elektronika, 11 (1962).

12. Krasnogorov, S.I.,"Combined evaluation of amplitude, phase, range and its derivatives by radar methods", Radiotekhnika i elektronika, 1 (1964).

13. Levin, B.R., "Theoretical bases of statistical electronics", Sov. radio (1968).

14. Lee, R.,"Optimum evaluations, definition of characteristics and control", Sov. radio (1968).

15. Linnik, Yu. V., Metod naimen'shikh kvadratov i osnovy teorii obrabotki nablyudenii [Method of least squares and bases of a theory for processing observations], Moscow, 1962.

16. Maryamov, A.T., Yastrebov, V.D., "A system of cylindrical coordinates for illustrating the motion of artificial satellites", Kosmicheskiye issledovaniya, 4, 5 (1966).

17. Middleton, D., "Introduction to the statistical theory of communication", Sov. radio (1962).

18. Olyanyuk, P.V., "Elements of the unified theory for measuring the parameters of motion of space vehicles", Kosmicheskiye issledovaniya, 6, 3 (1968).

19. Olyanyuk, P.V., Romanov, L.M., Mikhaylik, V.I., "Characteristics of defining different systems of orbital elements for space vehicles", Kosmicheskiye issledovaniya, 9, 1 (1971).

20. Rytov, S.M., "Introduction to statistical radiophysics", Nauka, Moscow, (1966).

21. Subbotin, M.F., Kurs nebesnoy mekhaniki [Course in Celestial Mechanics], Moscow, 1941.

22. Ustinov, B.A., "The motion of satellites in orbits having small eccentricity in the noncentral gravitational field of Earth", Kosmicheskiye issled., 5, 2 (1967).

23. Fal'kovich, S.Ye., "Reception of radar signals against a background of fluctuating interference", Sov. radio (1961).

24. Khelstrom, K. Statisticheskaya teoriya obnaruzheniya signalov [Statistical Theory of Signal Detection], Moscow, 1963.

25. Khoroshavtsev, V.G., Yastrebov, V.D., "Algorithms for defining the parameters of motion of AES using cylindrical coordinates", Kosmicheskiye issled., 4, 1 (1968).

26. Shevshayevich, V.S., "Introduction to the theory of space navigation", Sov. radio, (1971).

27. Teoreticheskiye osnovy radiolokatsii [Theoretical bases of radar], Ya. D. Shirman, Ed., Sov. radio, 1971.

28. Elyasberg, P. Ye., "Introduction to the theory of AES flight", Nauka, (1965).

29. Kelly, E. J., Wishner, RP., "Theory of a consisten filter for morgets moving at great speeds", IEEE Transactions on Military Electronics, MIL-9, 1, (1965).

30. Moe, Kenneth, "On optimum selection of a smoothing interval for computing orbits of artifical satellites", In: Proc. of the First Internat. Symp. on the Use of Art. Sat. for Geodesy, 1962.

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16. Abstract

The optimum reception of signals from space vehicles and artificial earth satellites is discussed in terms of potential accuracy of measurements. The parameters of motion are explained, and various illustrative mathematical formulas are used to compare a priori and actual data.

Topics covered include: autocorrelation functions, error correlation matrices, optimum signal filtration, orbital parameters, planetary radar systems, signal delay, signal processing, and various statistical methods of analysis.

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